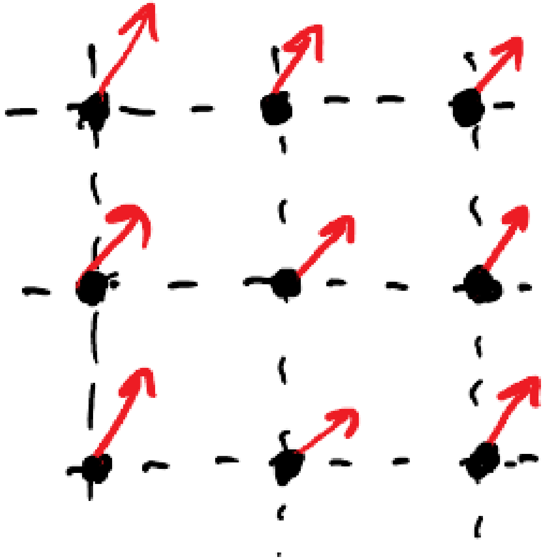
Detecting vortices with persistent homology

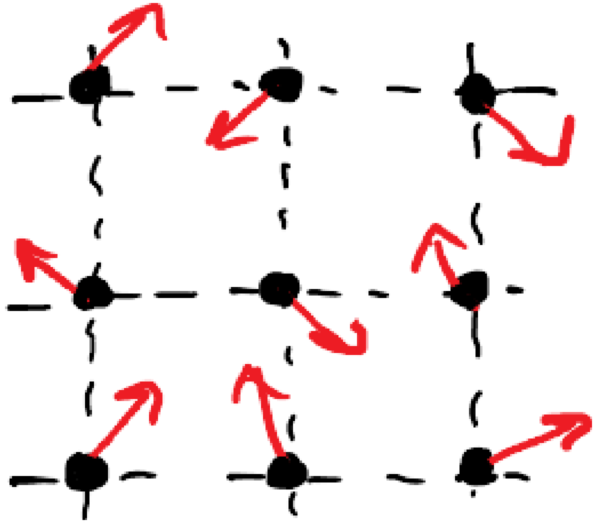
Centre for Topological Data Analysis Meeting 18th February 2022, Oxford

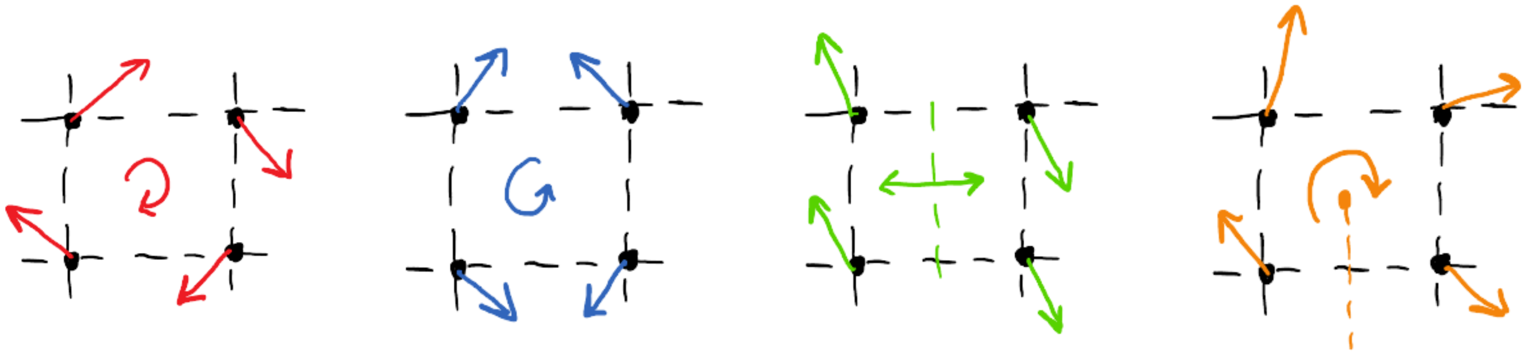
Nick Sale



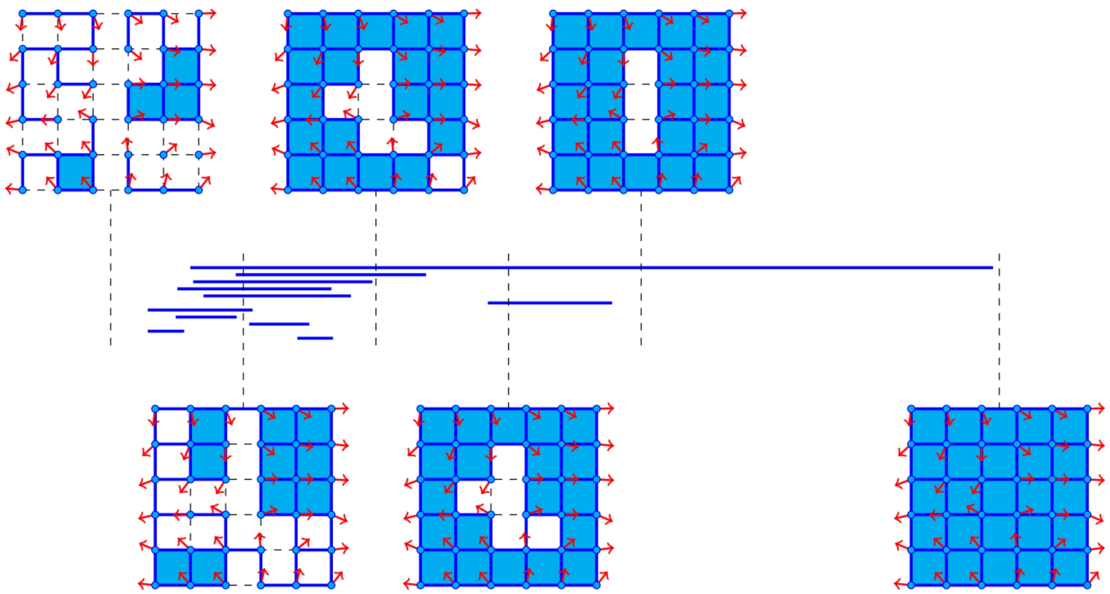
Increasing Temperature

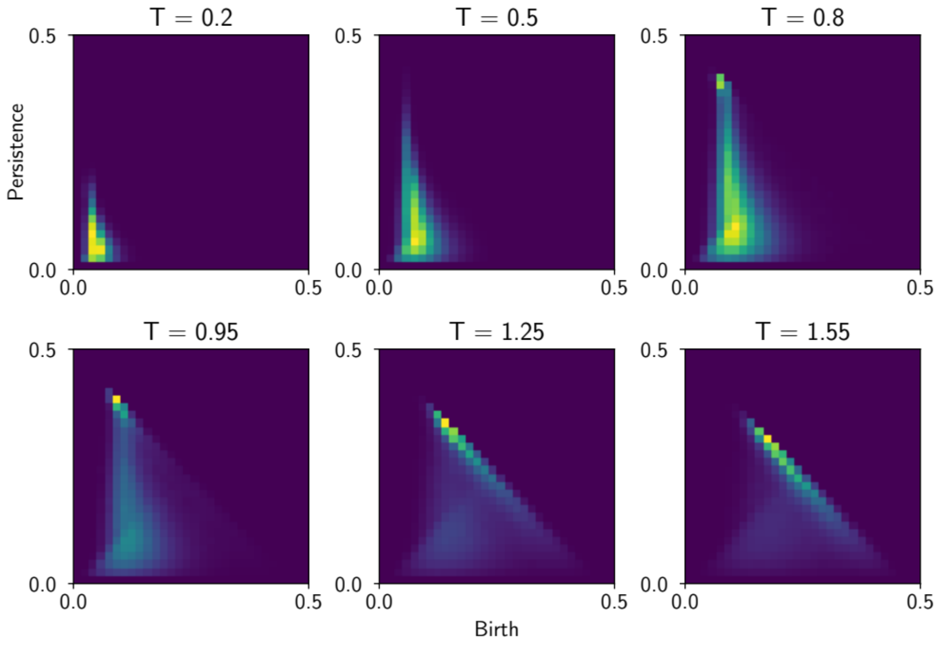
$$H(\boldsymbol{\theta}) = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$



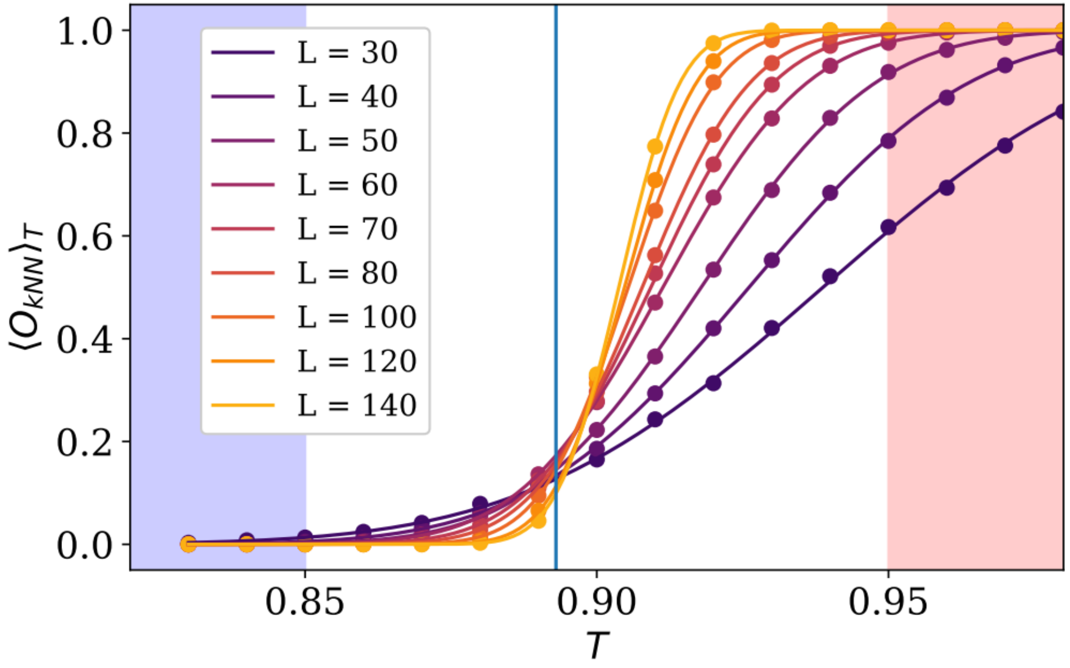


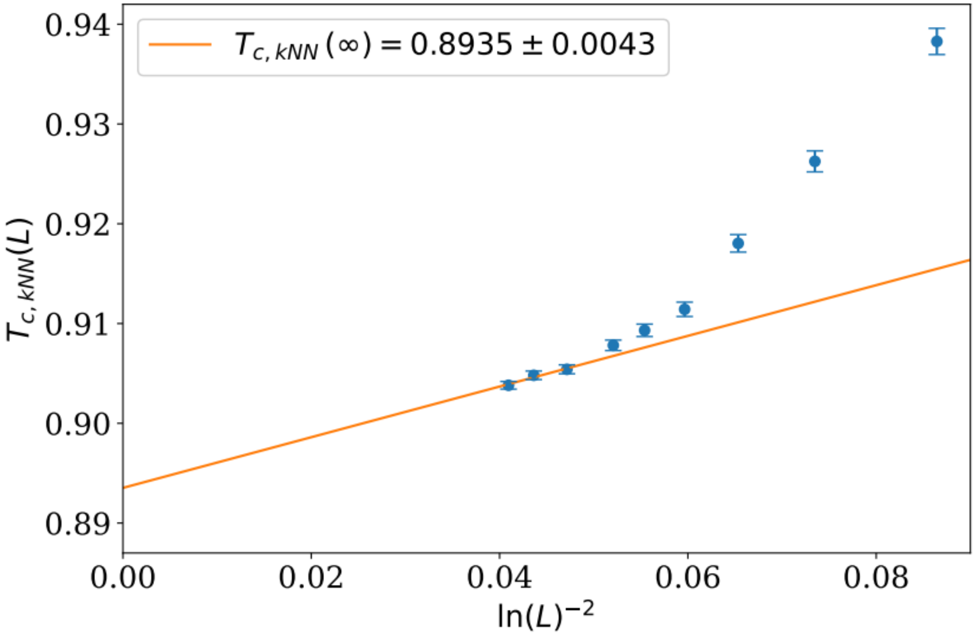
XY Models



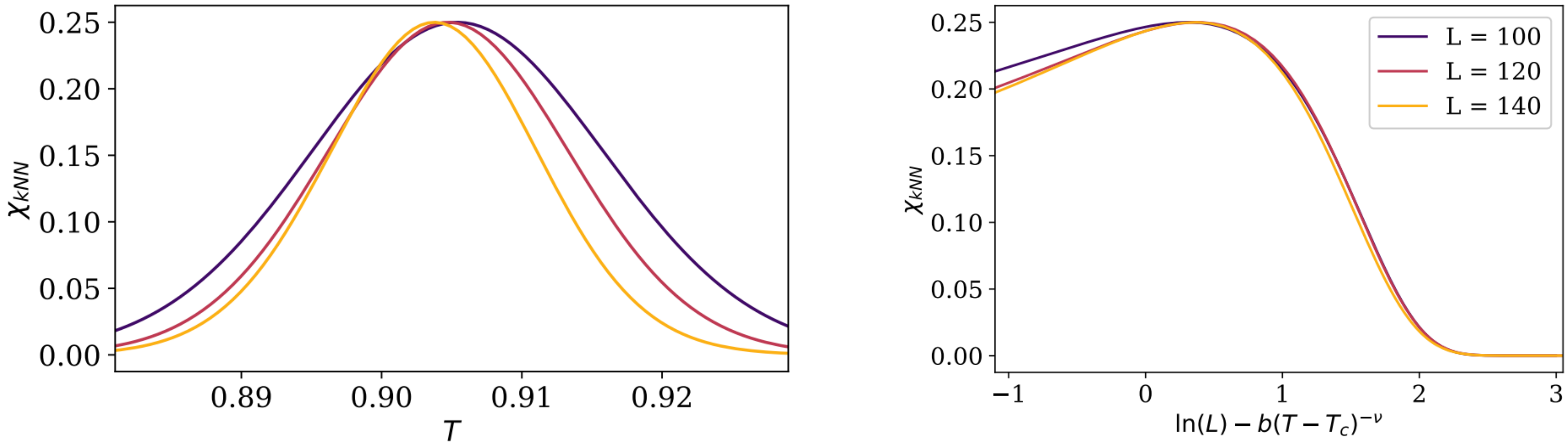


Persistence

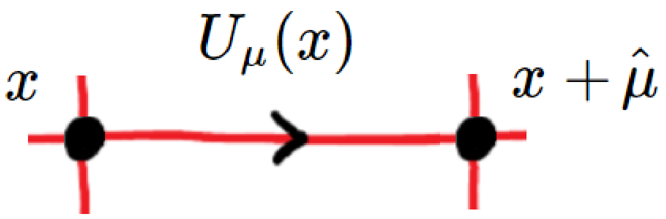




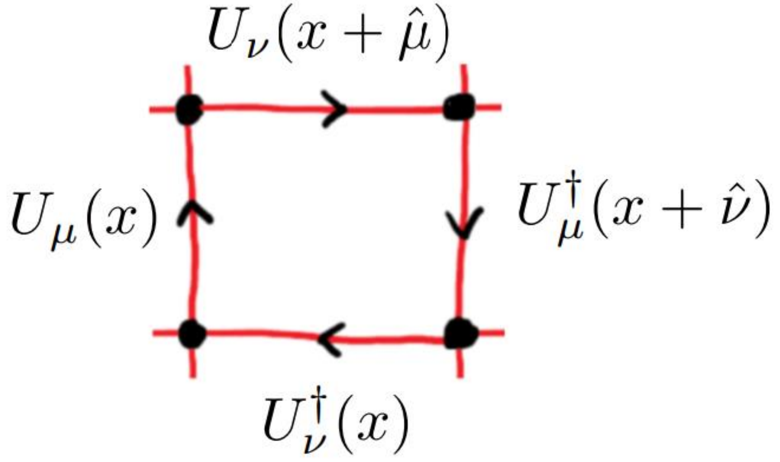
Critical Temperature



Critical Exponent



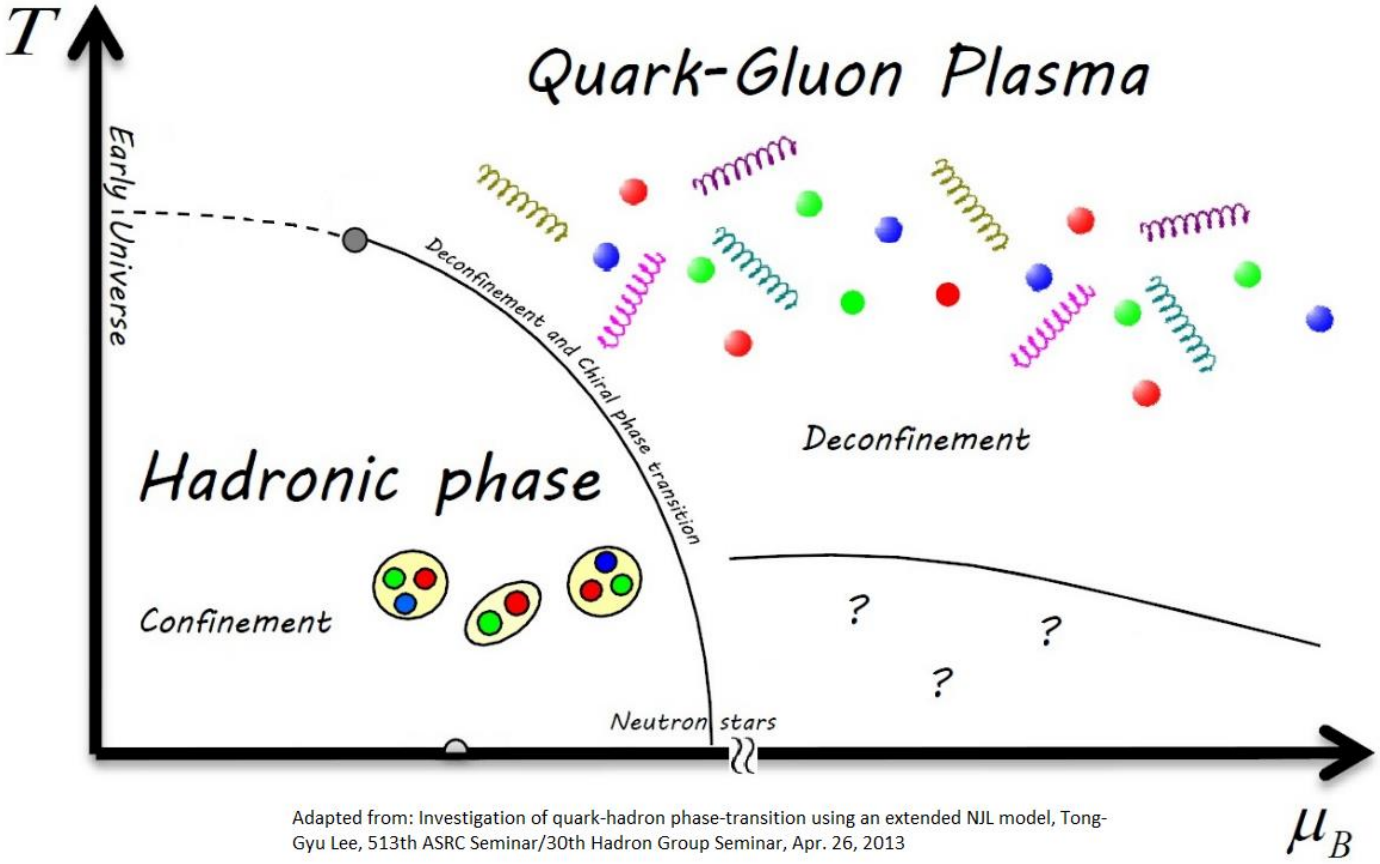
$$U_\mu(x) \to g(x) U_\mu(x) g^{-1}(x+\hat{\mu})$$



$$P_{x,\mu,\nu} = U_\mu(x)\, U_\nu(x+\hat{\mu})\, U_\mu^\dagger(x+\hat{\nu})\, U_\nu^\dagger(x)$$

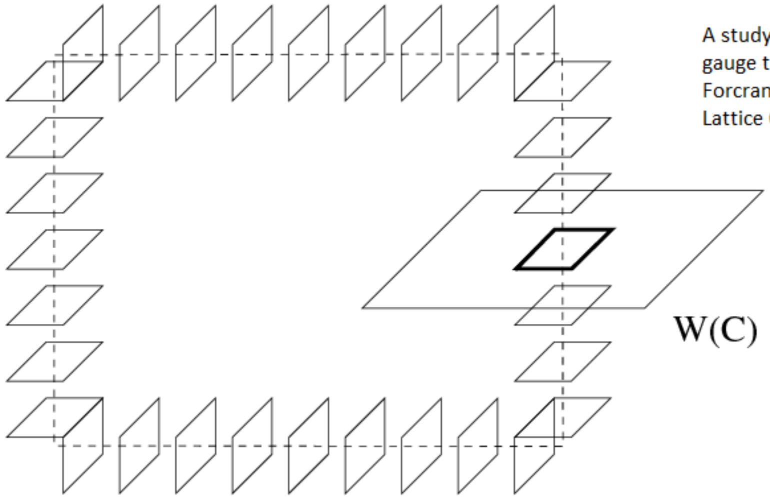
$$S(\mathbf{U}) = -\sum_{x,\mu,\nu} \frac{1}{2} tr(P_{x,\mu,\nu})$$

SU(2) Lattice Gauge Theory

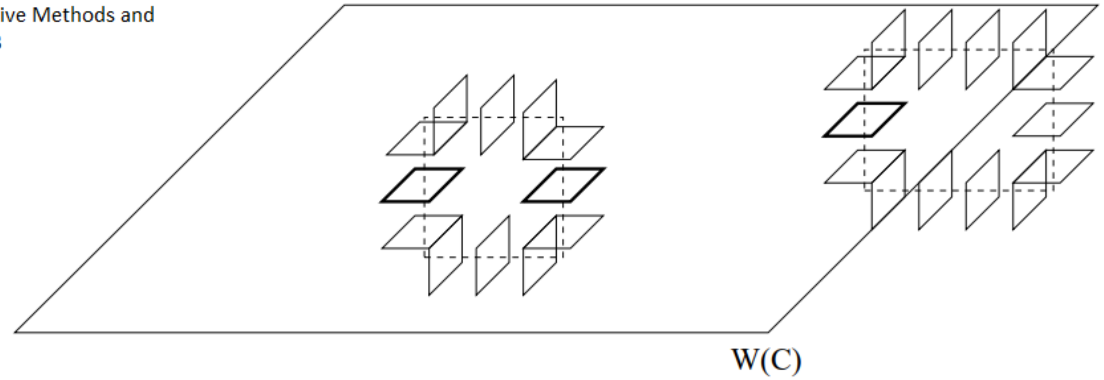


Adapted from: Investigation of quark-hadron phase-transition using an extended NJL model, Tong-Gyu Lee, 513th ASRC Seminar/30th Hadron Group Seminar, Apr. 26, 2013

(De)confinement

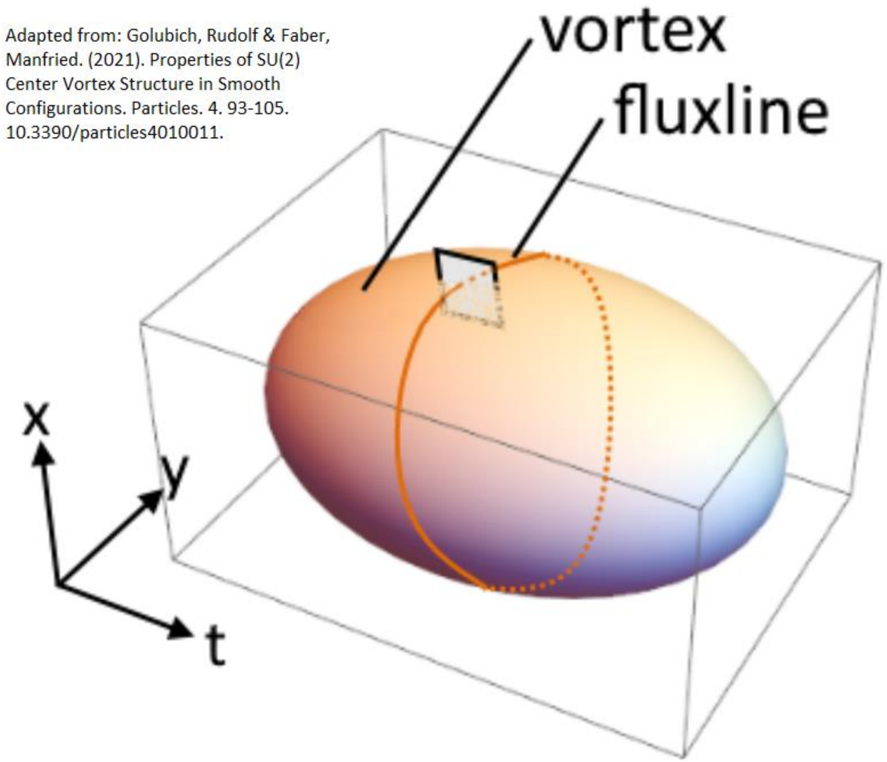


A study of center vortices in SU(2) and SU(3) gauge theories, Michele Pepe and Philippe de Forcrand, Non-Perturbative Methods and Lattice QCD, pp. 194-203

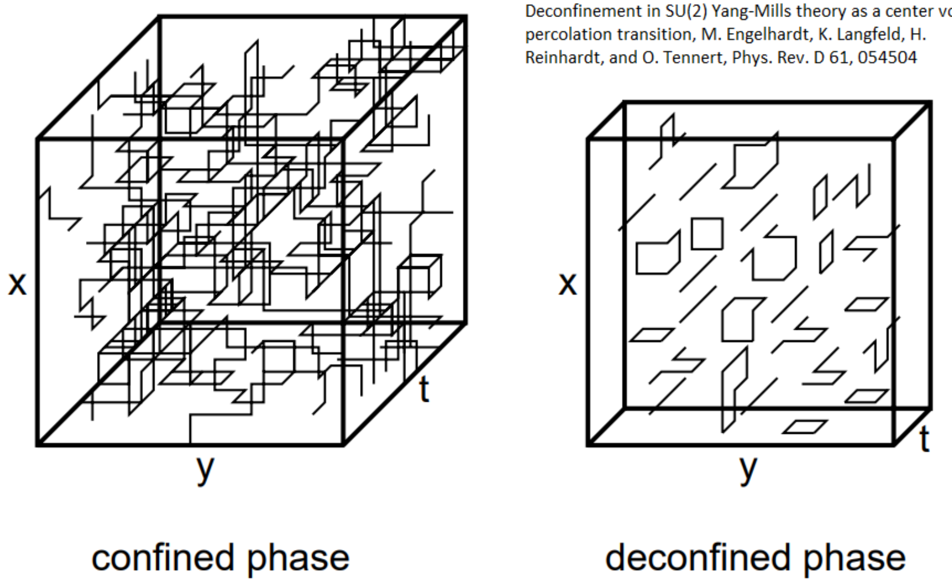


Area Laws, Perimeter Laws and Center Vortices

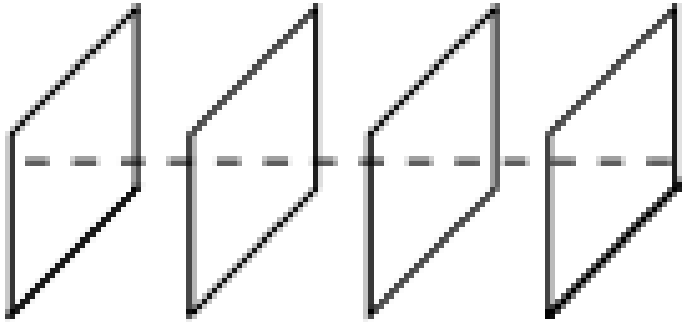
Adapted from: Golubich, Rudolf & Faber, Manfried. (2021). Properties of SU(2) Center Vortex Structure in Smooth Configurations. Particles. 4. 93-105. 10.3390/particles4010011.



Deconfinement in SU(2) Yang-Mills theory as a center vortex percolation transition, M. Engelhardt, K. Langfeld, H. Reinhardt, and O. Tennert, Phys. Rev. D 61, 054504



Center Vortices

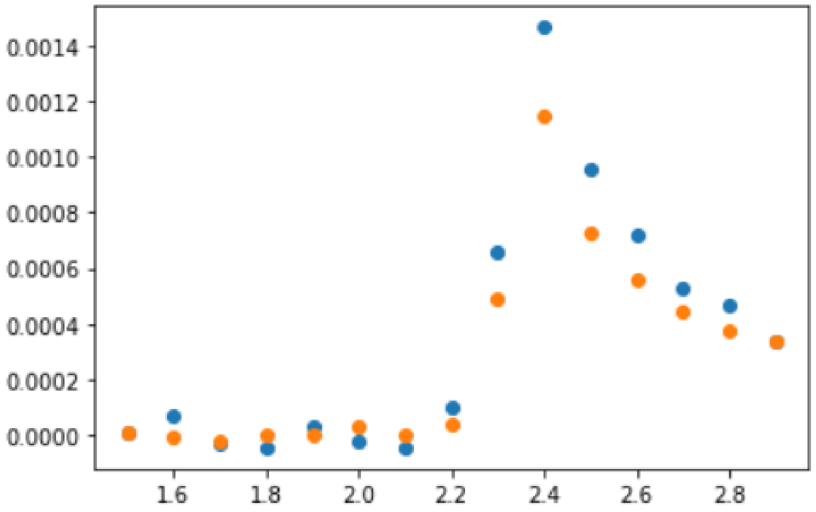


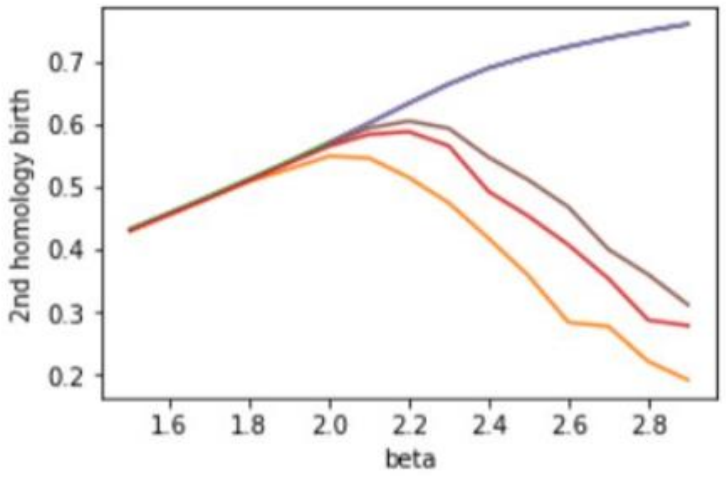
Idea: Filter "filled-in" dual lattice according to Wilson loop of plaquettes

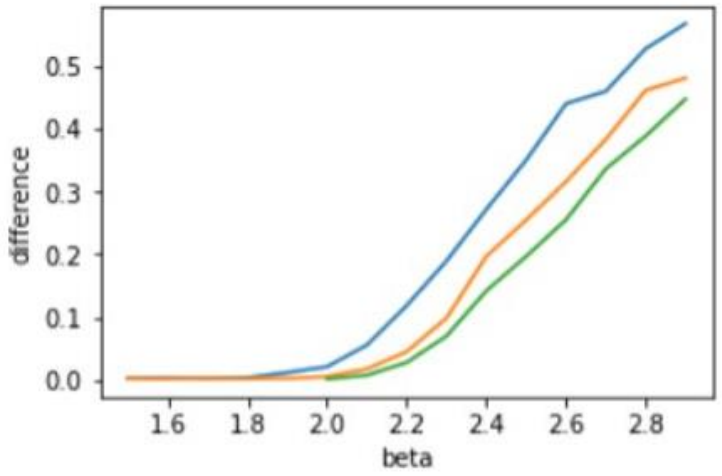
Introduce each plaquette (2-cell) at time equal to the WL of the plaquette it links with

Introduce 1-cells and 0-cells as needed

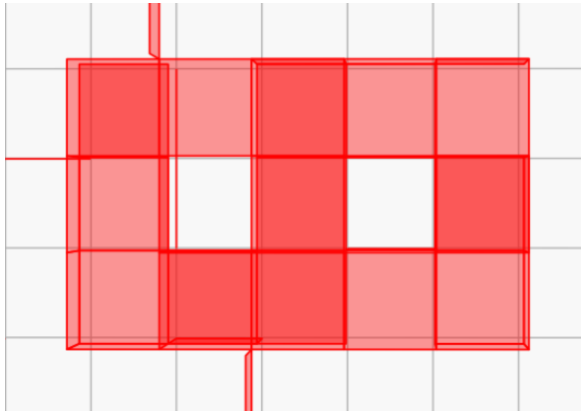
Introduce 3-cells and 4-cells introduced according to a "clique" rule

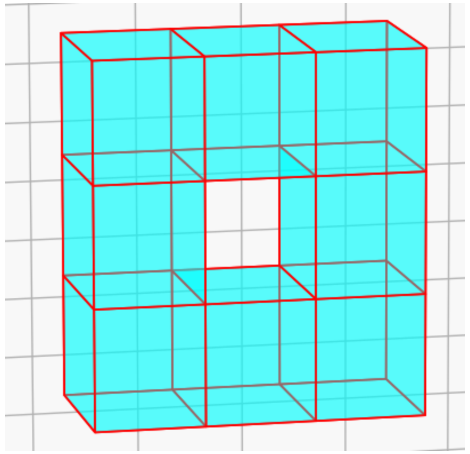


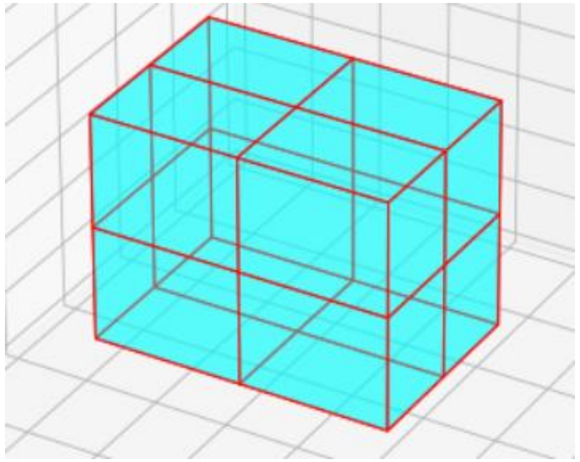


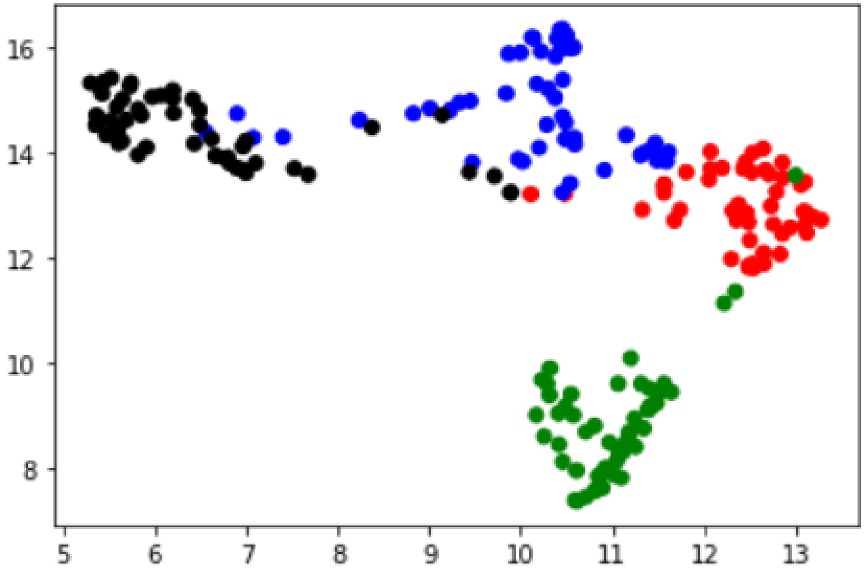


Twist Energy Order Parameter

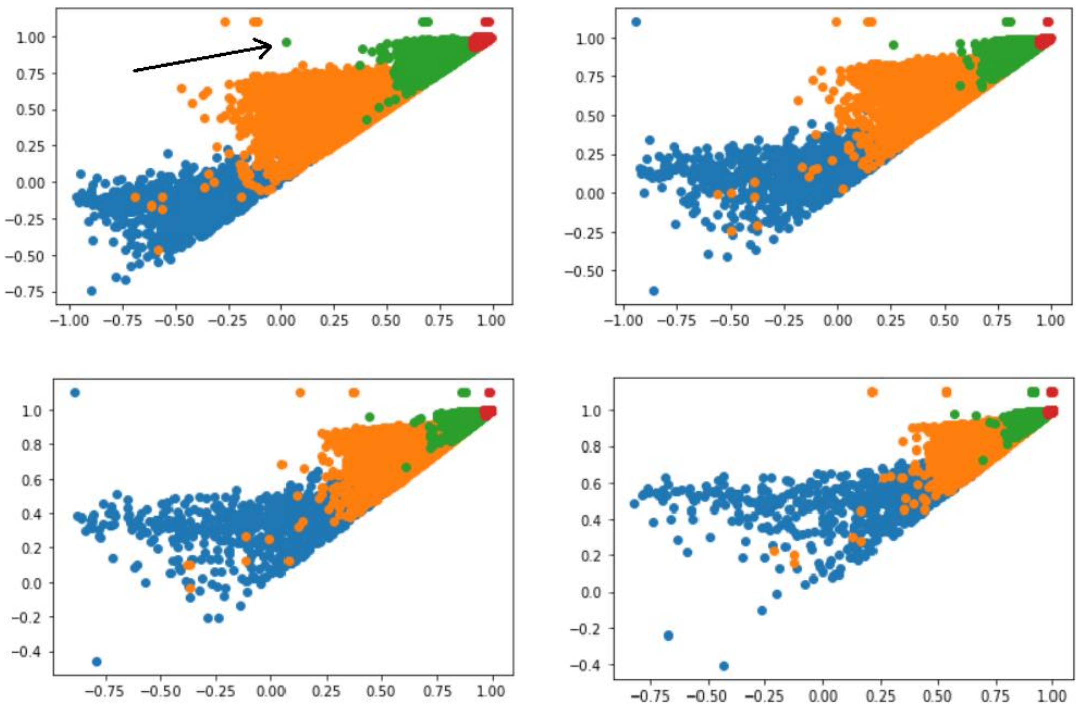


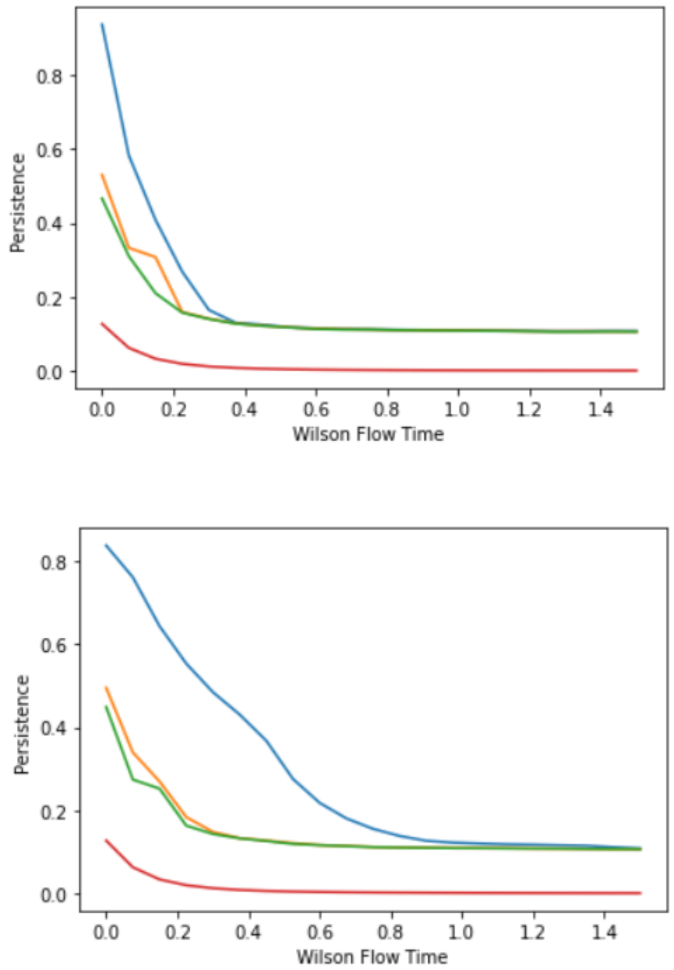




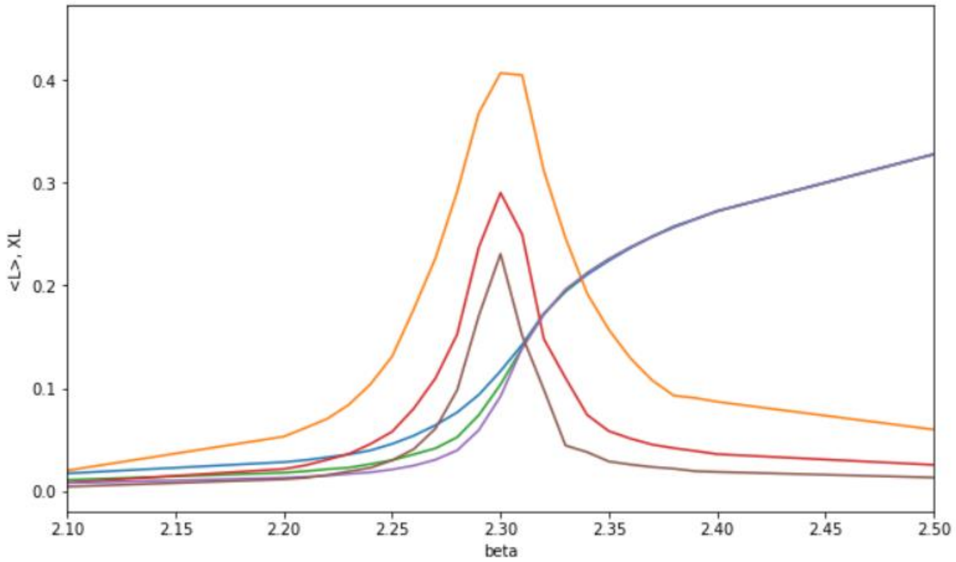


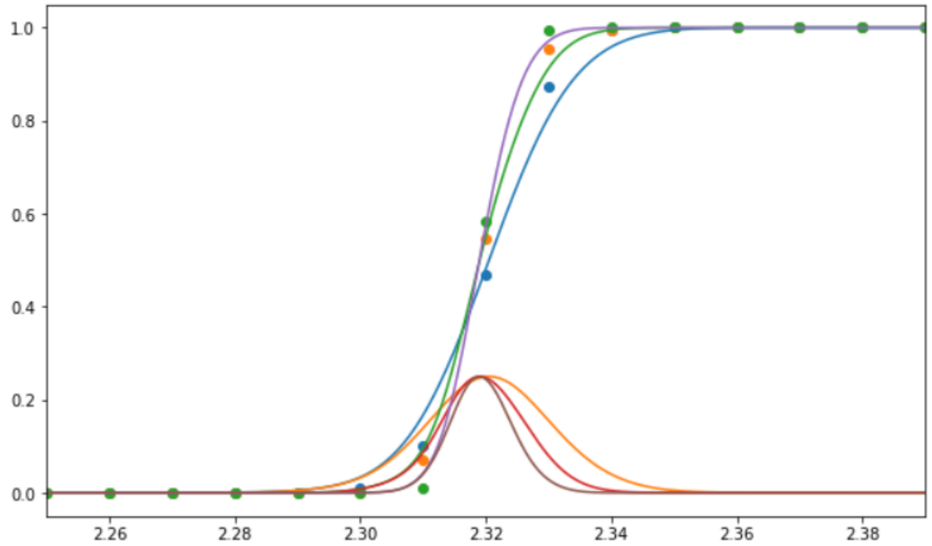
Distinguishing Vortex Surfaces





Physicality of Vortices





Quantitative Analysis of the Phase Transition

* Quantitative analysis of phase transitions in two-dimensional XY models using persistent homology, Nicholas Sale, Jeffrey Giansiracusa, Biagio Lucini, Phys. Rev. E 105, 024121

Summary

- Persistent homology lets us characterise vortices in XY models*
- It lets us probe vortices in SU(2) lattice gauge theory

Ongoing problems

- Detecting thick vortices directly
- Moving to SU(3) and introducing matter fields

Summary and Ongoing Work