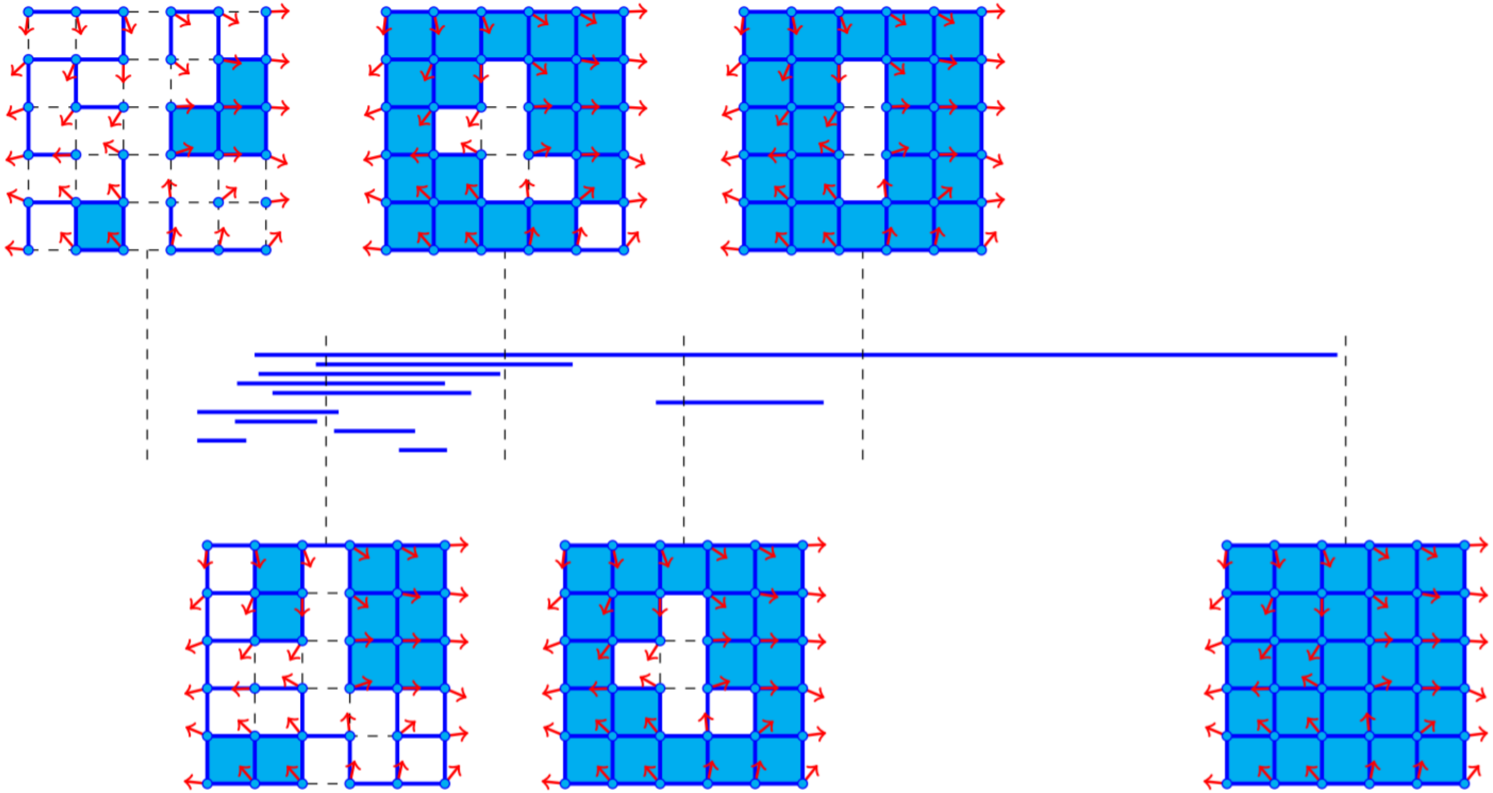
PERSISTENT HOMOLOGY AND PHASE TRANSITIONS



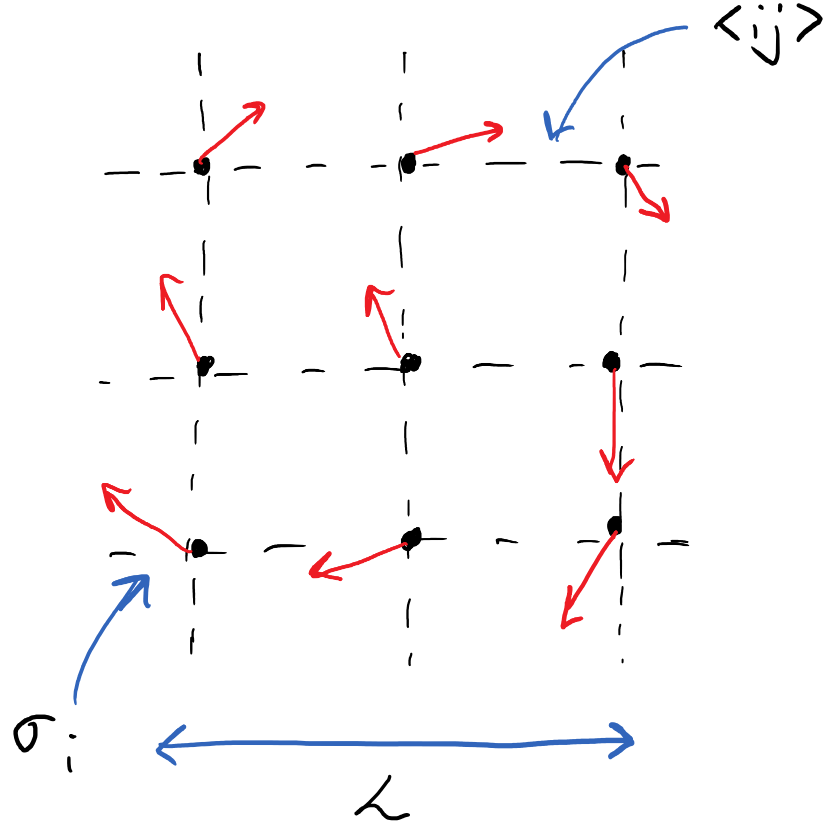
TopFlavours June 2021

Nick Sale

Swansea University

LATTICE SPIN MODELS

Example 2D XY Model



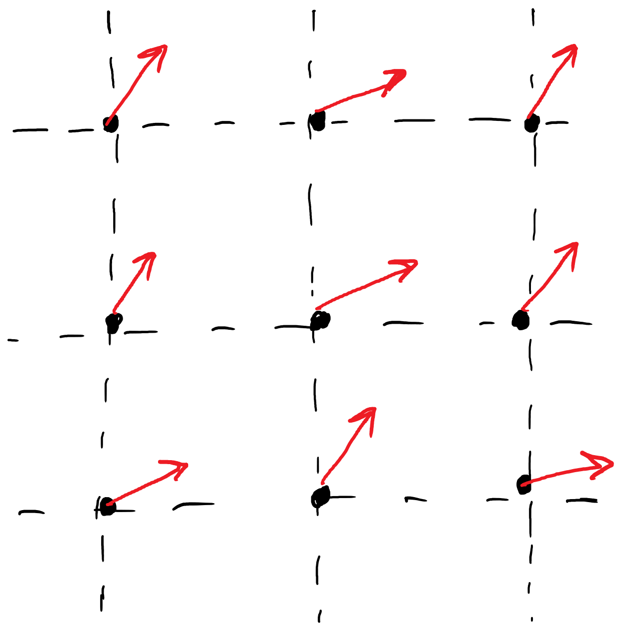
- $L \times L$ square lattice Λ
- $\sigma_i \in \mathbb{S}^1 \subseteq \mathbb{R}^2 \quad \forall i \in \Lambda$
- Configuration space $\mathcal{C} = (\mathbb{S}^1)^\Lambda$
- Hamiltonian $H : \mathcal{C} \to \mathbb{R}$

$$\{\sigma_i\}_{i \in \Lambda} \longmapsto -\sum_{\langle ij \rangle} \sigma_i \cdot \sigma_j$$

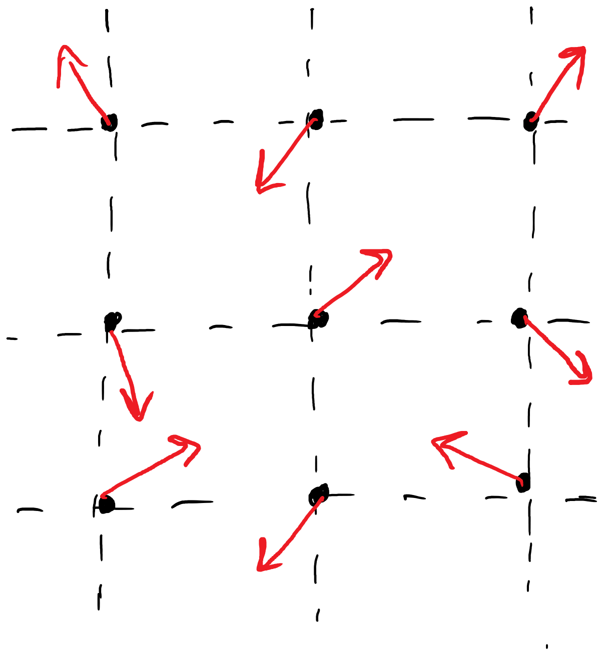
- $\mathrm{Prob}(x) \propto \exp(-\frac{1}{T} H(x))$

LATTICE SPIN MODELS

- Hamiltonian $\{\sigma_i\}_{i\in\Lambda} \longmapsto -\sum_{\langle ij\rangle} \sigma_i \cdot \sigma_j$
- $\mathrm{Prob}(x) \propto \exp(-\frac{1}{T} H(x))$

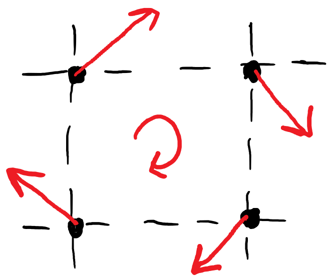


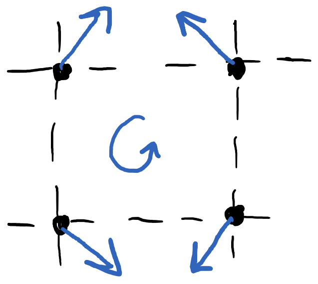
low temperature



high temperature

MOTIVATION

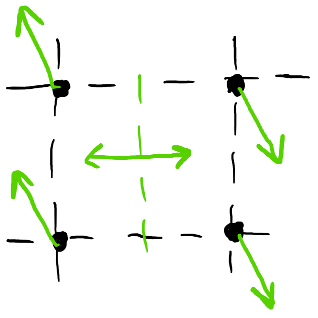


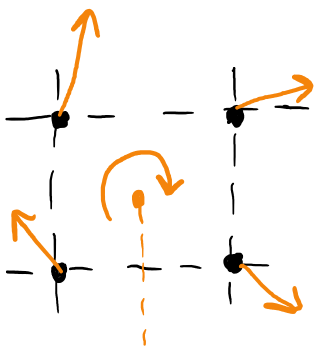


- Need for new observables
- Topological defects
  + geometric features

E.g.

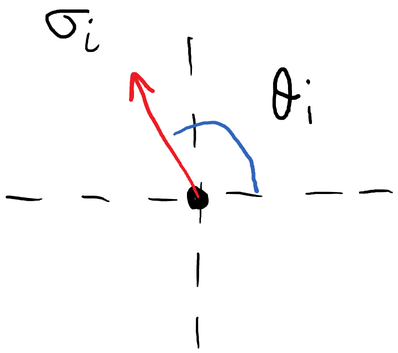
- Vortex – antivortex pairs
- Domain walls + half vortices

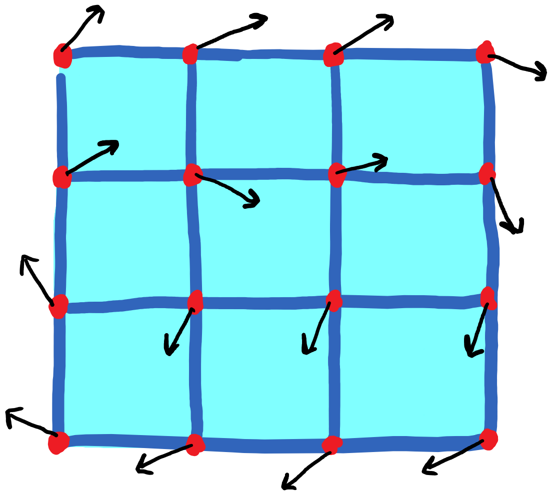




PERSISTENT HOMOLOGY

- Filtration $\mathcal{F} : \mathbb{R} \longrightarrow$ Cubical Complex
- $\mathcal{F}(r) = f^{-1}((-\infty, r])$





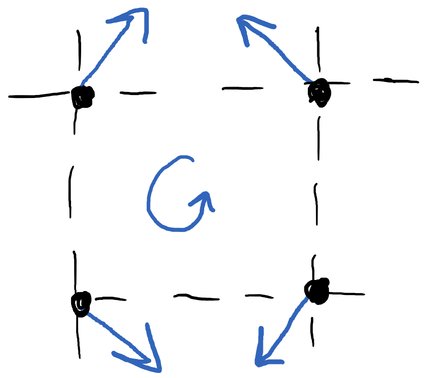
$f(\bullet) = 0$

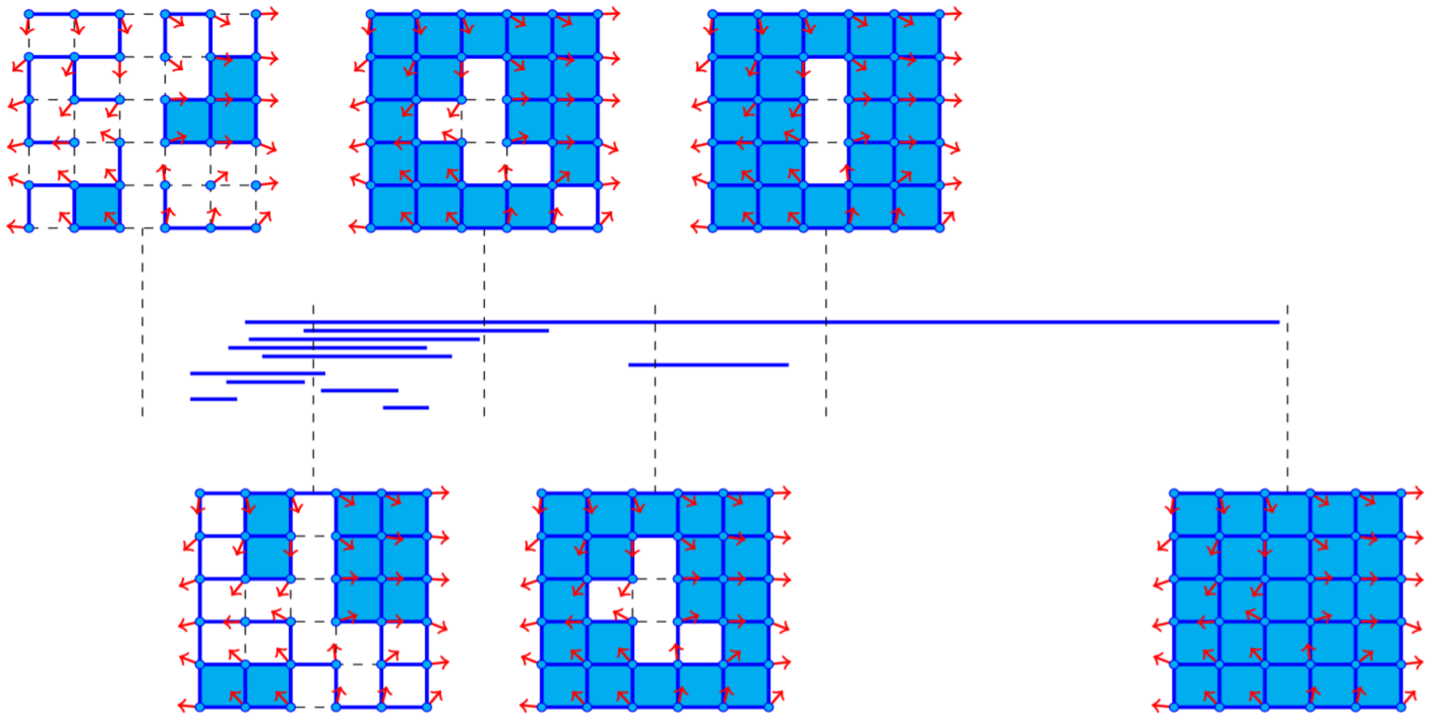
$f(\text{—}) = |\theta_i - \theta_j|$

$f(\blacksquare) = \max_{ij \in \square} \{ |\theta_i - \theta_j| \}$

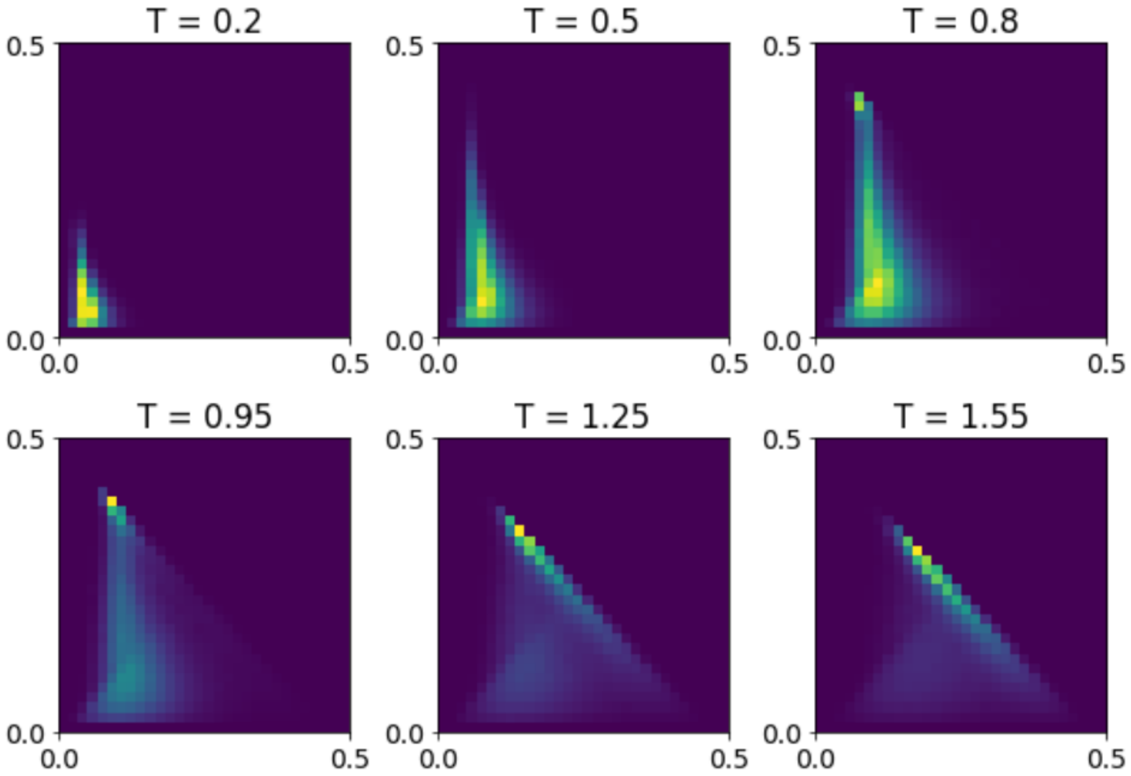
PERSISTENT HOMOLOGY

- Apply H_1 and track generators





PERSISTENT HOMOLOGY



PHASE CLASSIFICATION