Probing center vortices and deconfinement in SU(2) lattice gauge theory with persistent homology

Nick Sale, Biagio Lucini,
Swansea University

Jeff Giansiracusa
Durham University

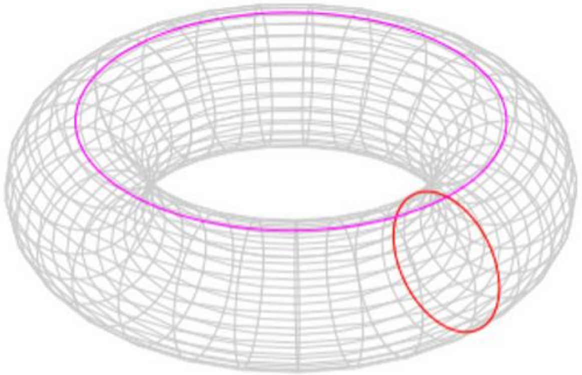
Lattice 2022, University of Bonn, 8th August

Outline

- Persistent homology
- Looking for center vortices in 4D SU(2) LGT without gauge fixing
- Quantitative analysis of the deconfinement phase transition

Homology

- k-th homology H_k is a map $H_k : Top \to Vect$
- k = 0 counts connected components
- k > 0 counts k-dimensional holes
- E.g. $\dim(H_0(X)) = 1$, $\dim(H_1(X)) = 2$, $\dim(H_2(X)) = 1$
- It is functorial: $f : X \to Y$ induces $f_* : H_k(X) \to H_k(Y)$

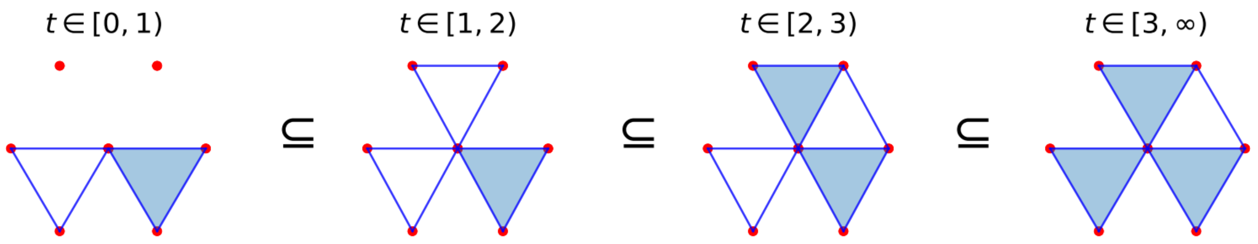


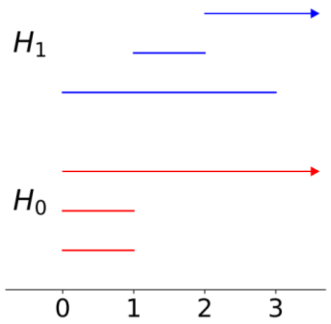
Persistent homology

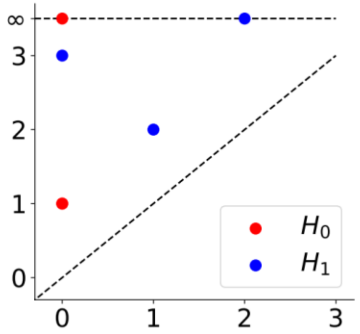
- Idea:

data → filtered complex → topological summary

Application Specific — Persistent Homology

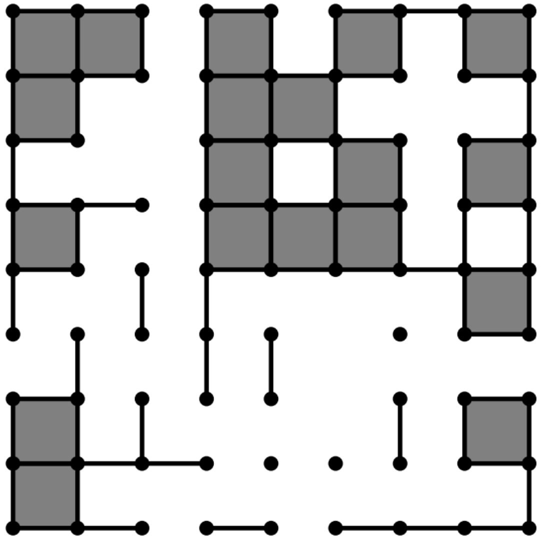






Cubical Complexes

- Topological spaces made of cubes of different dimensions
- Specify a filtration by giving time f(c) at which each cube c enters

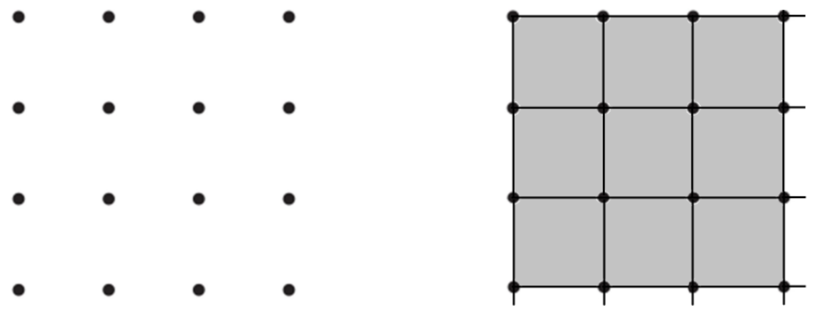


- Cubes must enter after their boundaries

Werman, Wright, Intrinsic Volumes of Random Cubical Complexes, Discrete & Computational Geometry volume 56, pp93–113 (2016)

Lattice as a Cubical Complex

- A square lattice induces a natural cubical complex
- E.g. a 2D lattice (with periodic boundary conditions)



- We use the complex corresponding to the dual of a 4D lattice

A Filtered Complex for Center Vortices

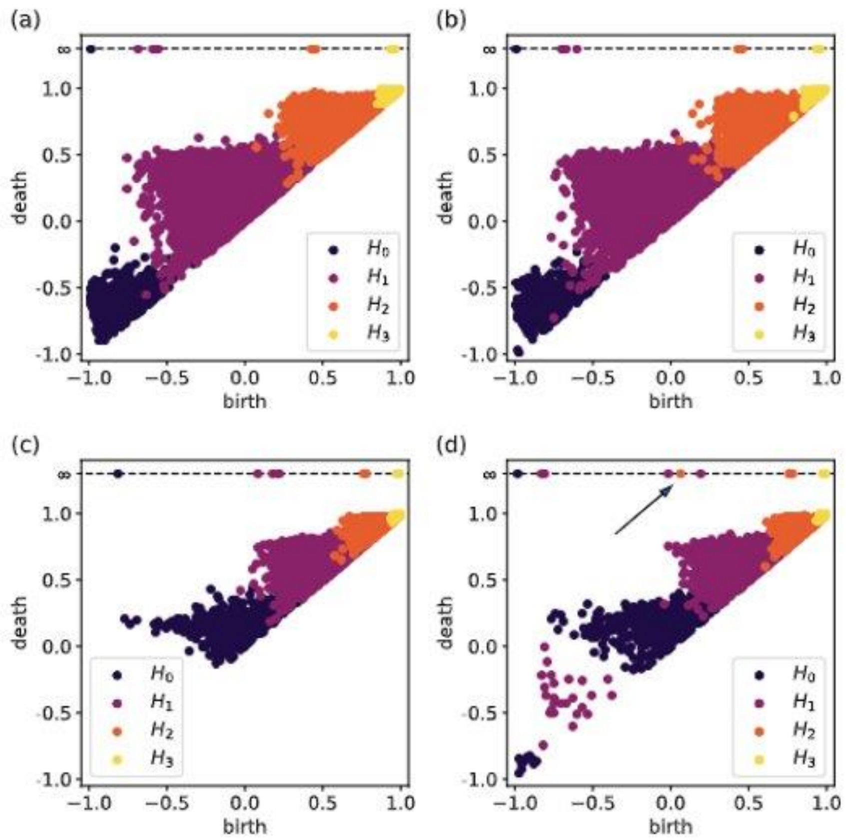
- Idea: explicitly construct vortex surfaces early in the filtration
- A 1x1 plaquette ☐ in the dual lattice links with the boundary of exactly one 1x1 plaquette ◇ in the lattice
- In the filtration, set f(☐) = Wilson loop around ◇
- For points, edges c: f(c) = min(f(☐) | c in ☐)
- For 3-cubes, 4-cubes c : f(c) = max(f(☐) | ☐ in c)

Motivation

- If vortex surface is thin
  - 1x1 Wilson loops linking with vortices are negated
  - Vortex surface enters filtration early, filled in late
  - Detect as a persistent point in PH_2 (assuming orientable)
- If vortex surface is thick
  - 1x1 Wilson loop multiplied by a partial rotation, still lowering the trace
  - Vortex surface enters filtration less early, filled in late
  - Detect as a less persistent point in PH_2

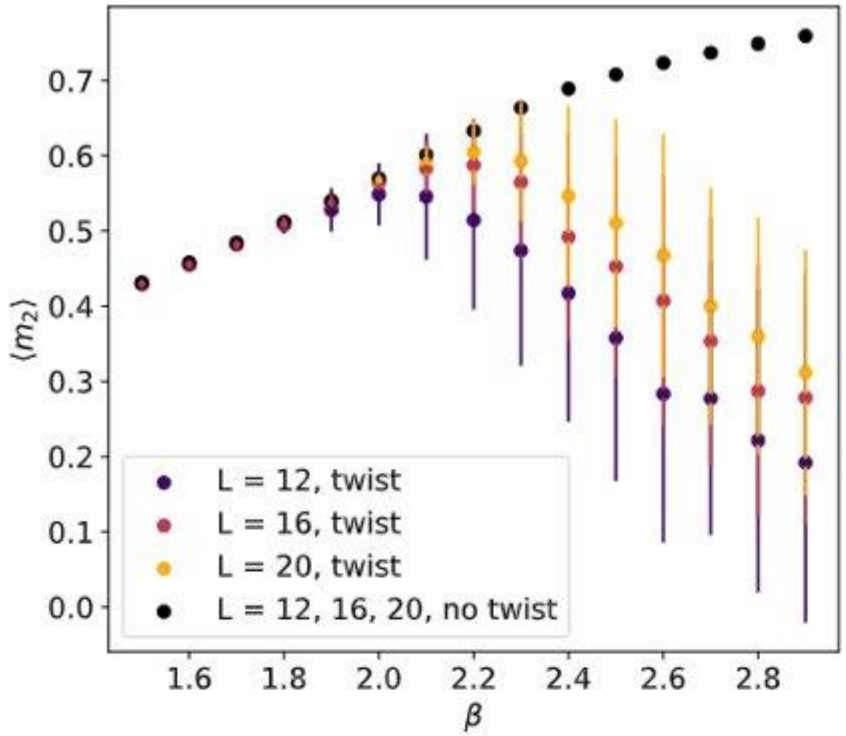
Twisted Boundary Conditions

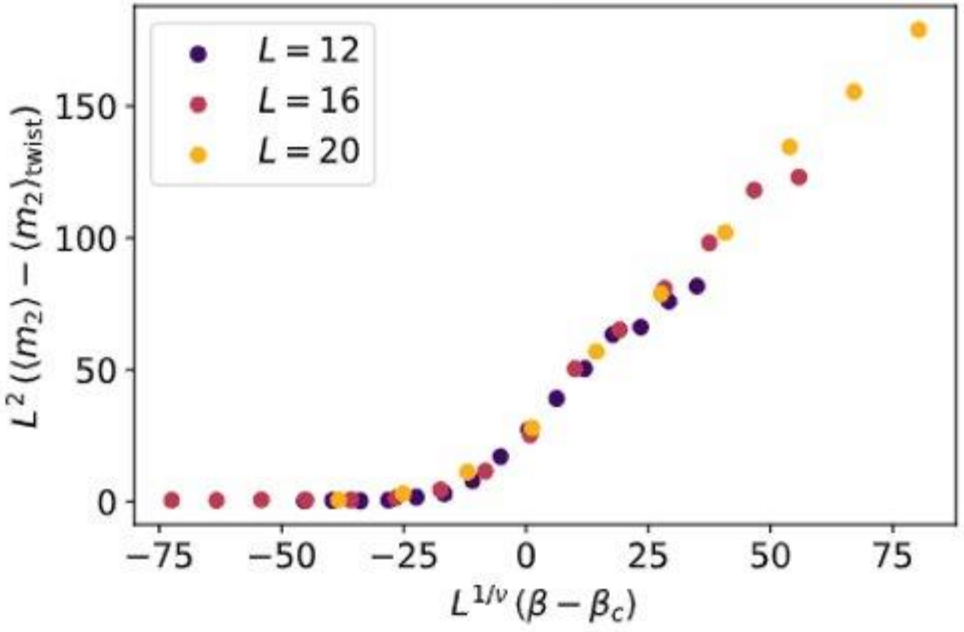
- Insert a vortex that wraps the periodic boundary
- In deconfined phase we see a particularly persistent point in H_2
- $m_2 = \min(b \mid (b, \infty) \text{ in } PH_2)$



Twisted Boundary Conditions

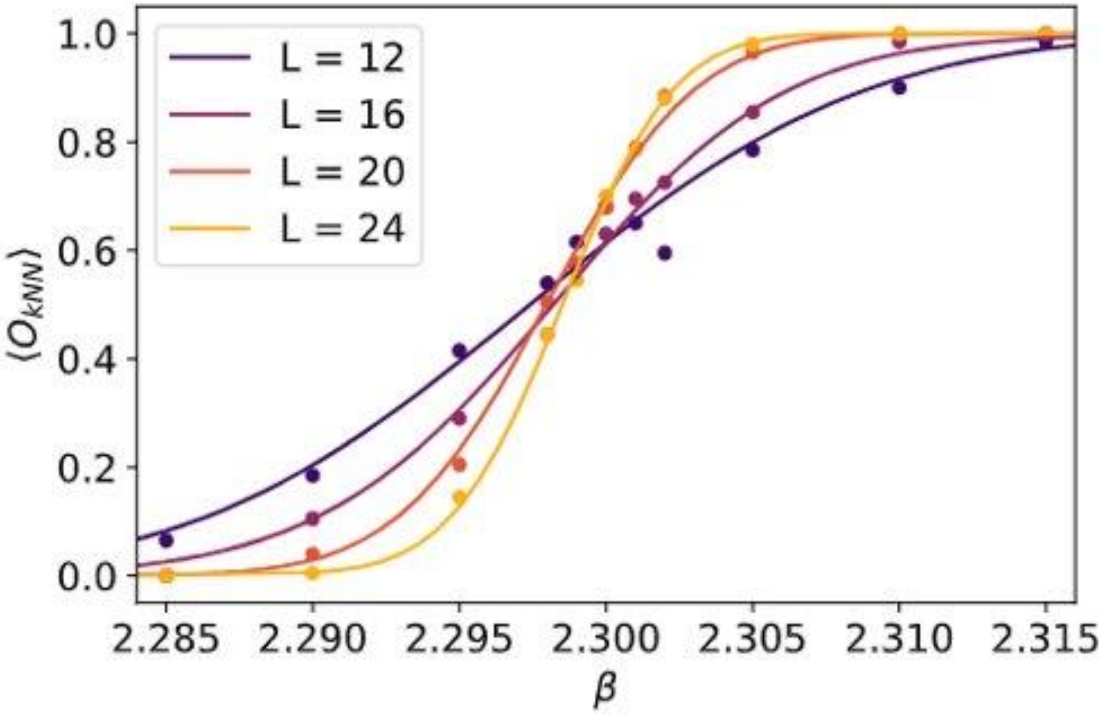
- Finite size scaling of $\langle m_2 \rangle$ - $\langle m_2 \rangle_{twist}$ gives good estimate of β_c and ν

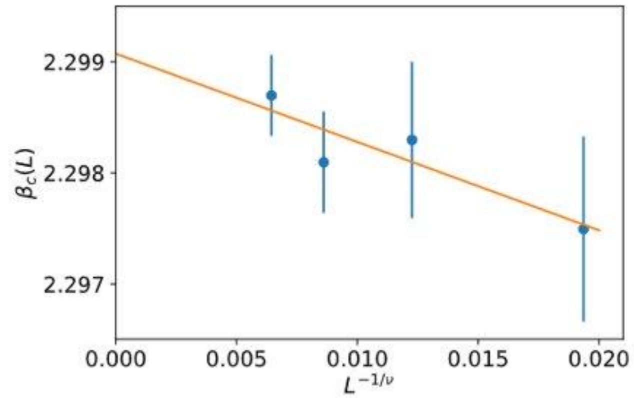


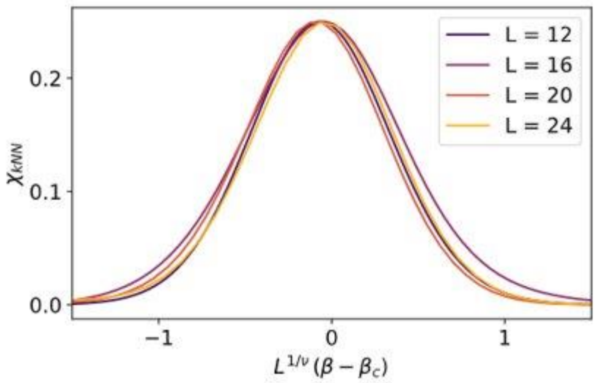


Without Twisted Boundary Conditions

- Apply k-NN classifier to guess phase from persistence diagram
- Estimates β_c and ν for N_t = 4,5,6







Outlook

- Persistent homology is a useful tool for lattice field theory
  - Here we looked at center vortices
  - But others have applied it more generally (Sehayek + Melko, Spitz + Urban + Pawlowski)

- Evidence for center vortex picture?

Thanks!

- Interested? Come say hi

- Or read the paper on the arXiv: *Probing center vortices and deconfinement in SU(2) lattice gauge theory with persistent homology*