Detecting vortices with persistent homology

Young Topologists Meeting
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Nick Sale, Swansea University

What's the idea?

- Apply TDA to statistical physics / quantum field theory
- Topological defects / objects in lattice simulations
- New tools might help provide insight in areas like Quantum Chromodynamics (QCD)

Outline

- Framework of statistical physics (and QFT simulations)
- 2D XY Model
- 4D SU(2) Lattice Gauge Theory
- Outlook

Statistical Physics

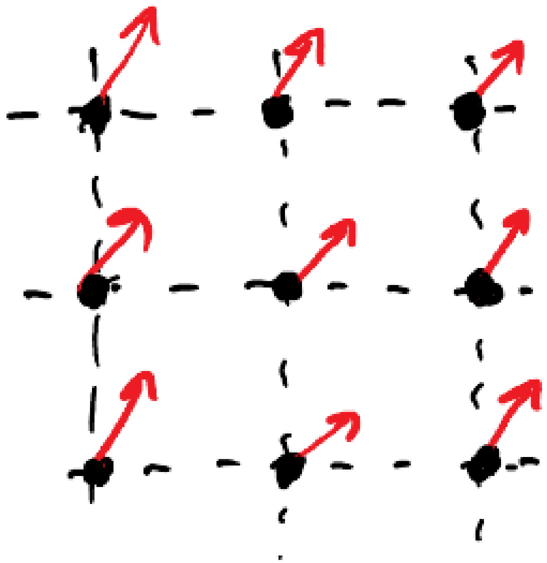
- A model is
  - A space of configurations $\mathcal{C}$
  - A Hamiltonian (called the action in QFT) $\mathcal{H} : \mathcal{C} \to \mathbb{R}$
- Probability dist. over $\mathcal{C}$ given by $$Pr(c) = \frac{1}{Z(\beta)} \exp(-\beta \mathcal{H}(c))$$
- Expectations $$\langle O \rangle := \mathbb{E}[O] \approx \frac{1}{N} \sum_{i=0}^{N} O_i$$

Phase Transitions

- Many ways to characterise, but let's keep it vague
- As we vary β a phase transition is a point at which the qualitative behavior of the model changes

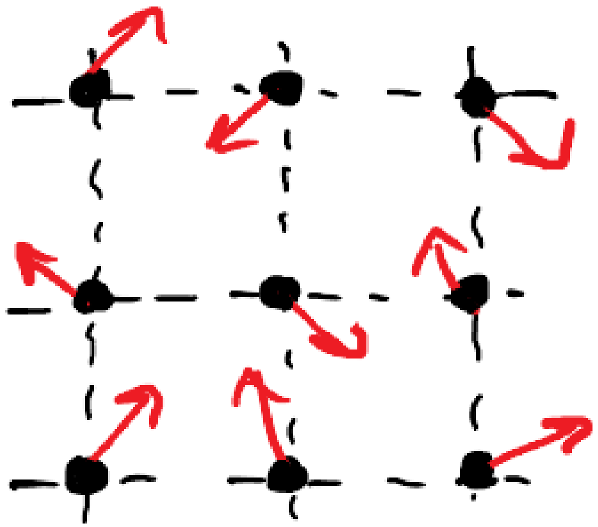
2D XY Model

- Configurations $\theta : \mathrm{verts}(\Lambda) \to S^1$
- Hamiltonian $\mathcal{H}(\theta) = -\sum_{\langle ij \rangle} \cos(\theta(i) - \theta(j))$



Increasing Temperature

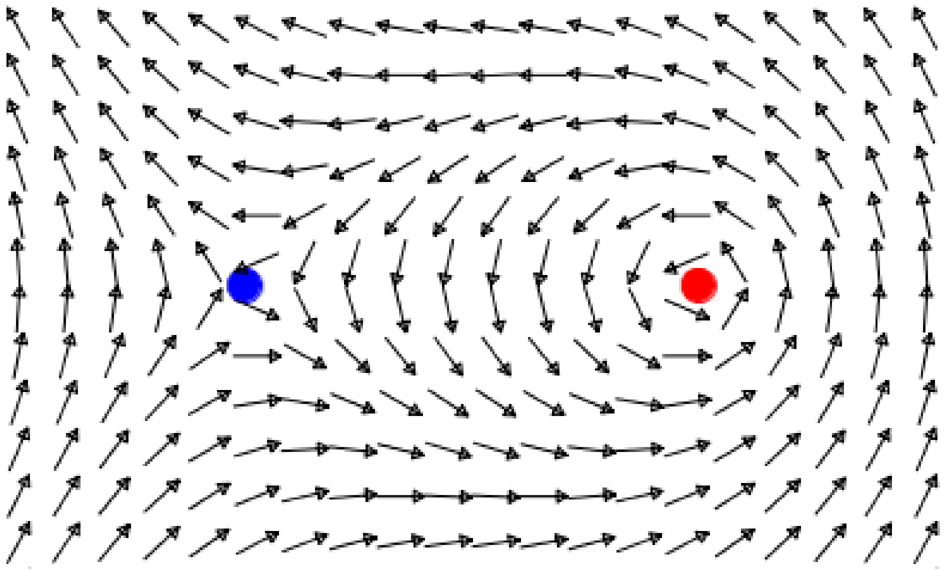
$$T = \frac{1}{\beta}$$



BKT Transition

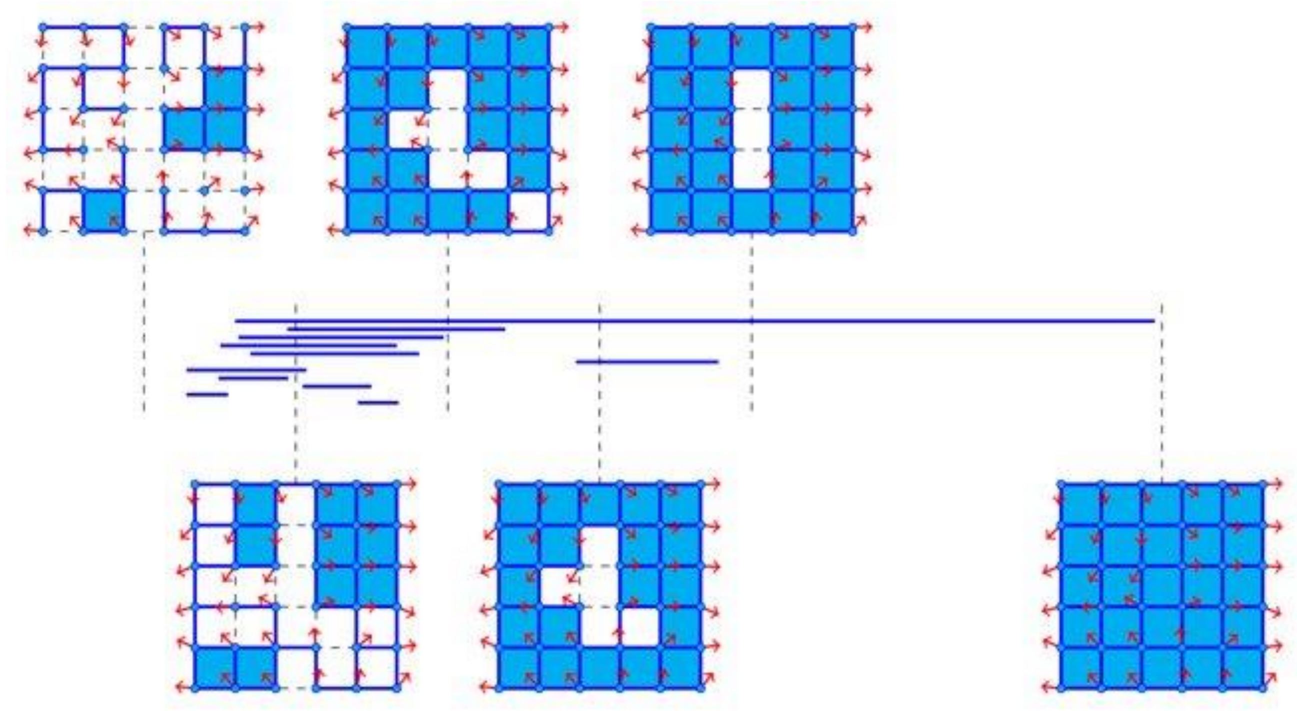
- Qualitative change at $T_c = 0.893$: correlation of spins across distance

- Driven by vortices and antivortices



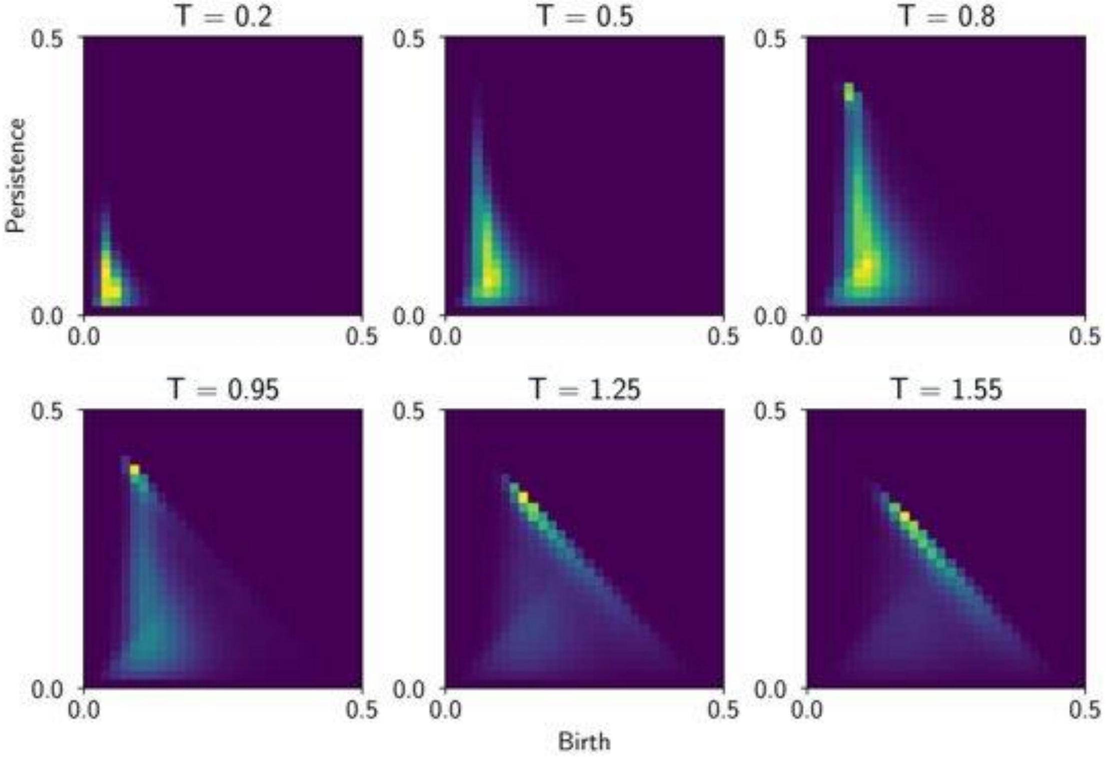
Persistent Homology of the XY Model

- Idea: filter the tiling of the 2-torus corresponding to the lattice according to difference in neighbouring spins



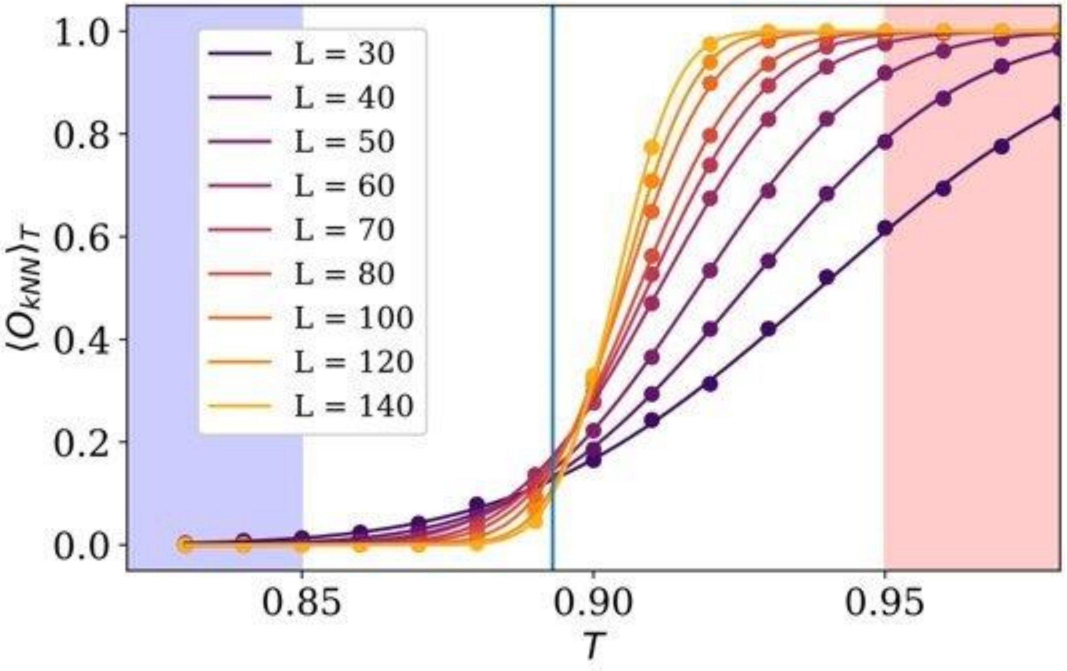
Can we see the phase transition?

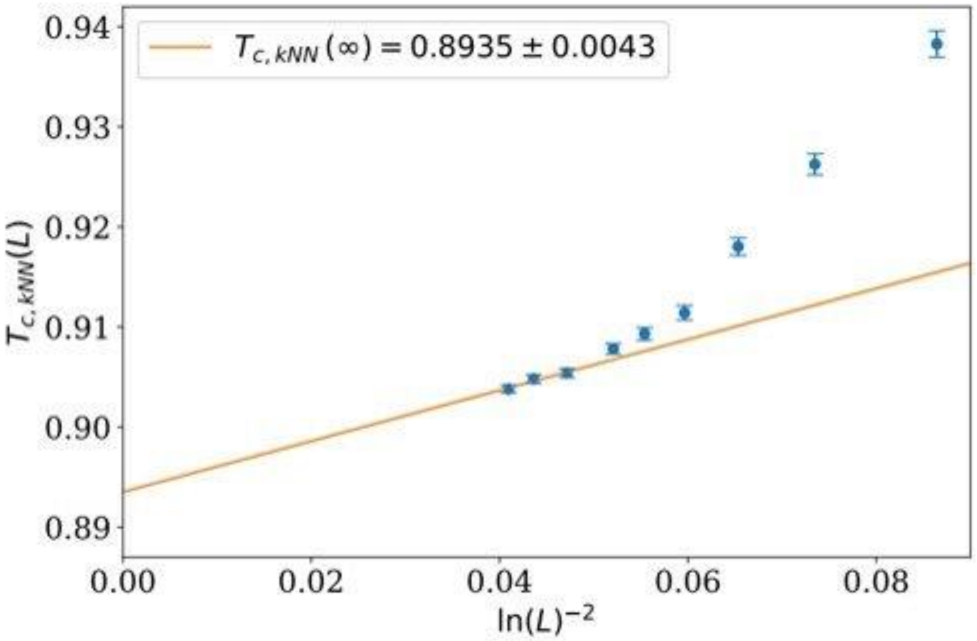
- Average persistence images (≈ density of persistence diagram)



Quantitatively?

- Train a k-nearest neighbour classifier on persistence images
- Use finite-size scaling to extract critical temperature



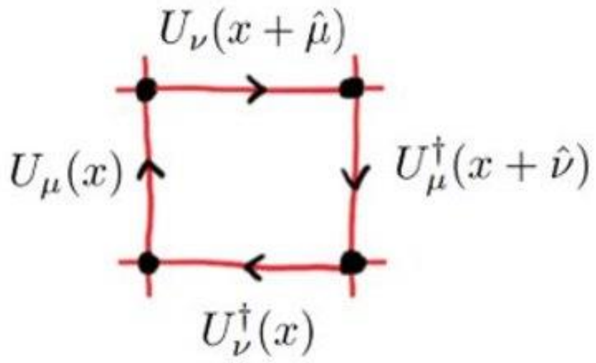


4D SU(2) Lattice Gauge Theory

- Configurations $U : \mathrm{edges}(\Lambda) \to \mathrm{SU}(2)$

- Gauge Invariance

$$W_{x,\mu,\nu} = tr\Big[U_\mu(x)\, U_\nu(x+\hat{\mu})\, U_\mu^\dagger(x+\hat{\nu})\, U_\nu^\dagger(x)\Big]$$



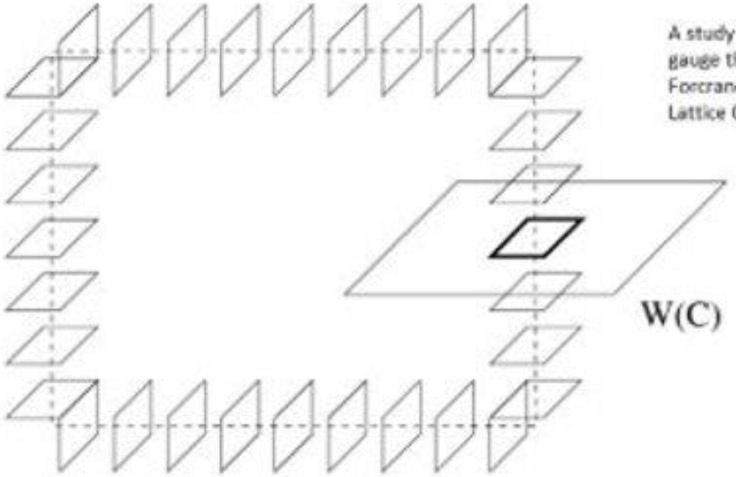
- Action $S(\mathbf{U}) = -\frac{\beta}{4} \sum_{x,\mu<\nu} W_{x,\mu,\nu}$

Deconfinement Transition

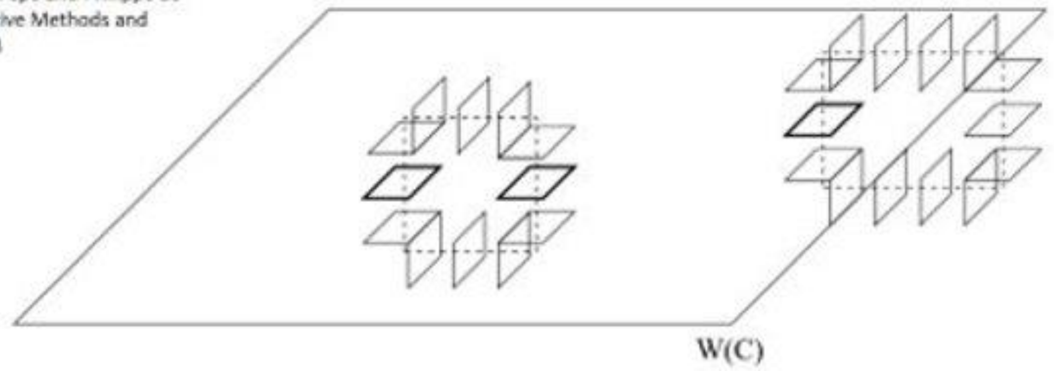
- Many characterisations, including area law for Wilson loops

$$\langle W(C)\rangle \sim \begin{cases} \exp(-\mathcal{A}(C)) & \beta \leq \beta_c \\ \exp(-\mathcal{P}(C)) & \beta > \beta_c \end{cases}$$

- Mechanism not known for sure: we look at the center vortex picture



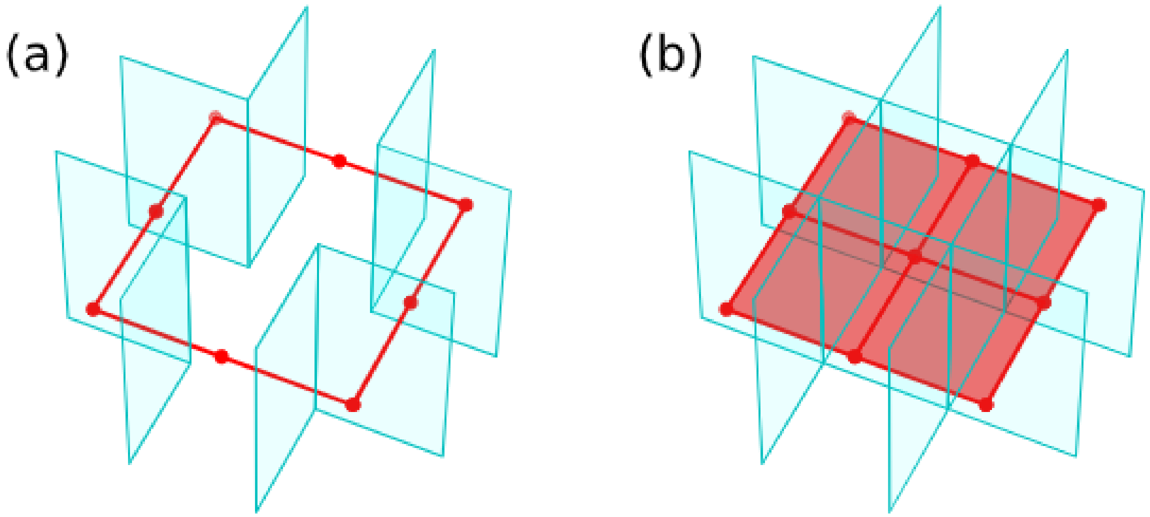
A study of center vortices in SU(2) and SU(3) gauge theories, Michele Pepe and Phillippe de Forcrand, Non-Perturbative Methods and Lattice QCD, pp. 194-203



Persistent Homology of SU(2) LGT

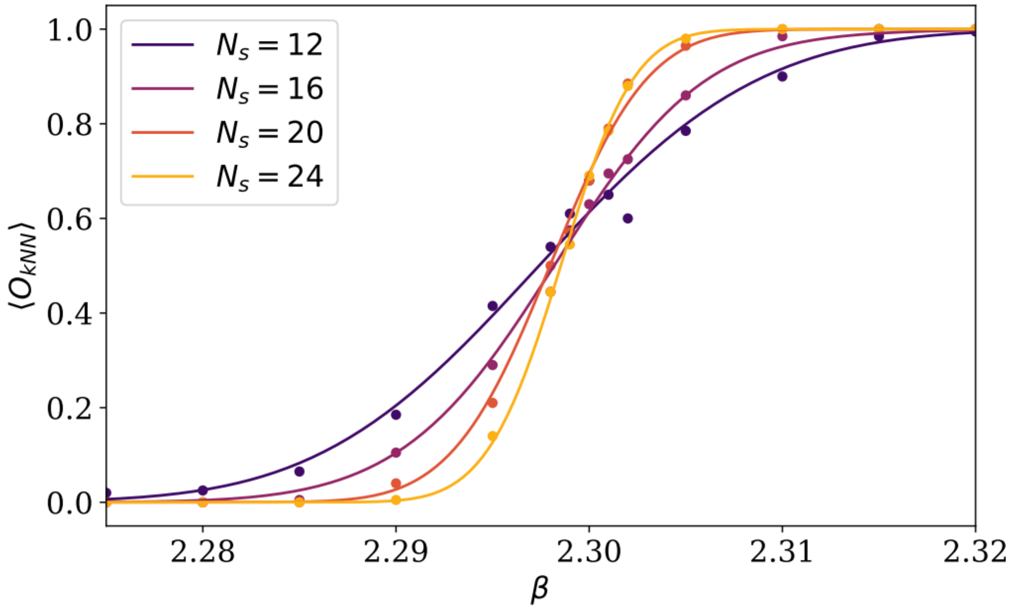
- Idea: filter the cubical tiling of the 4-torus corresponding to the dual lattice according to Wilson loop around plaquettes

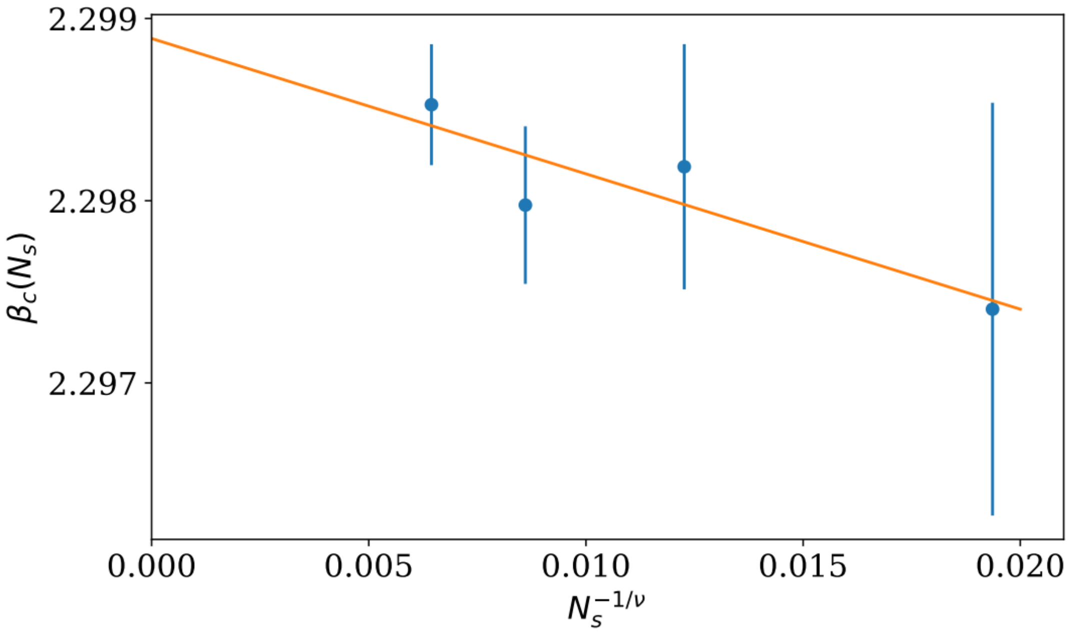
- Introduce each plaquette (2-cube) at time equal to the WL of the plaquette it links with



Quantitative Analysis

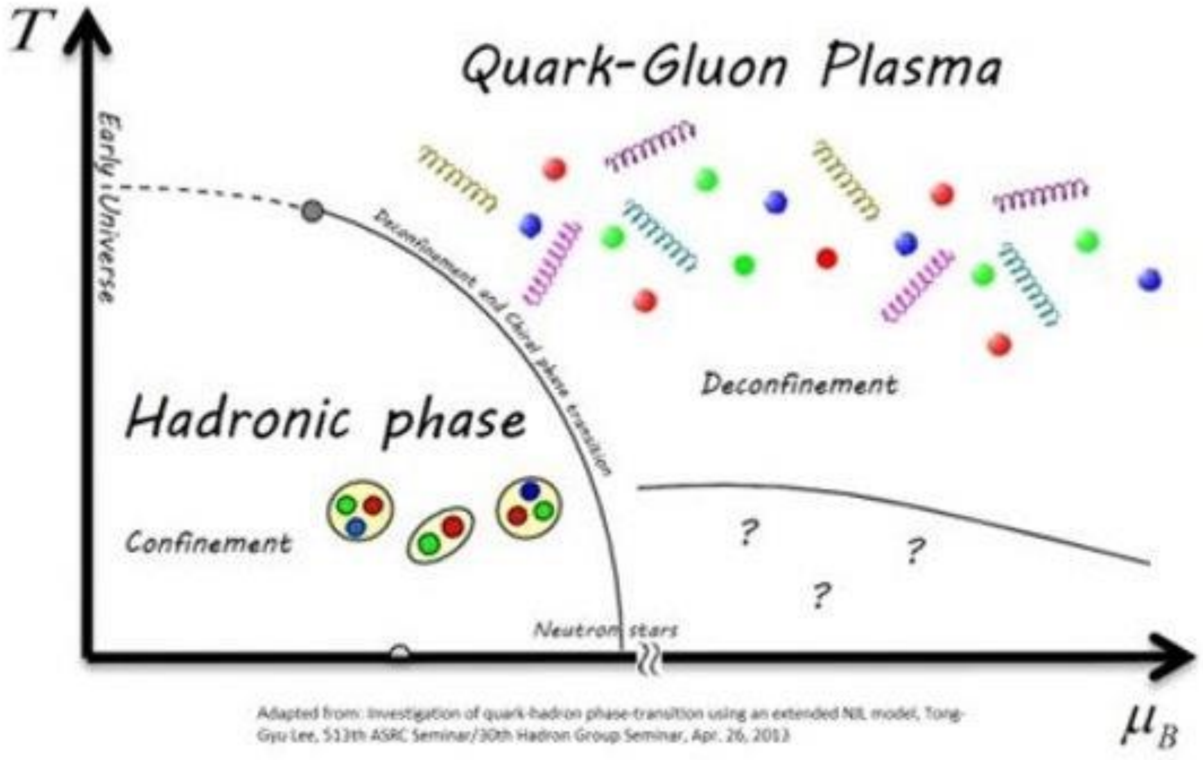
- Same idea as before: train kNN classifier and apply finite-size scaling





Outlook

- QCD



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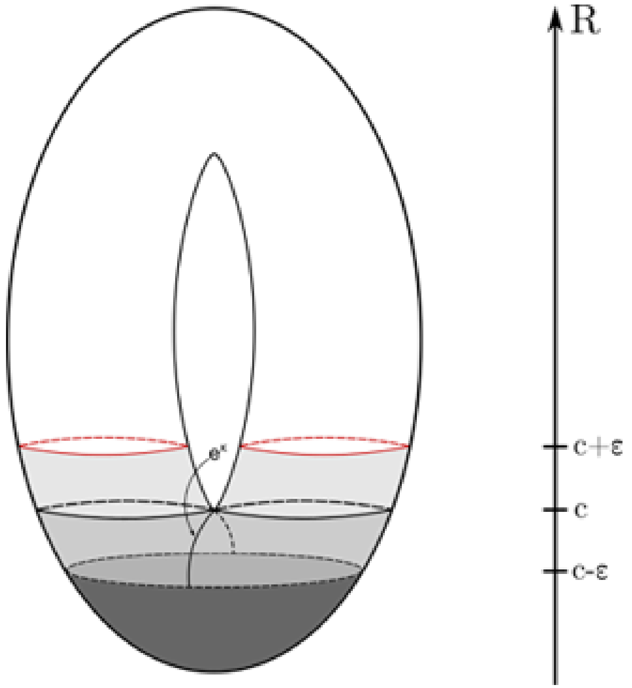
Quark-Gluon Plasma
Early Universe
Deconfinement and Chiral phase transition
Deconfinement
Hadronic phase
Confinement
?
?
?
Neutron stars
μ_B
Adapted from: Investigation of quark-hadron phase-transition using an extended NJL model, Tong-Gyu Lee, 513th ASRC Seminar/30th Hadron Group Seminar, Apr. 26, 2013

Outlook

- Persistent homology of configuration space approach

- Topology hypothesis for origin of phase transitions

$$\mathcal{C}_{\leq E} = \{c \in \mathcal{C} \,|\, \mathcal{H}(c) \leq E\}$$



Thanks!

- Interested? Come say hi or find the papers on arXiv

- To summarise:

  - PH lets us spot topological defects in simulation data

  - Quantitative nature lets us do quantitative analysis

  - Evidence for role of these defects in phase transitions?